PART 5a : Worked examples
Example to EN 1991 Part 1-2: Compartment fire

P. Schaumann, T. Trautmann
University of Hannover – Institute for Steel Construction, Hannover, Germany

1 TASK

The gas temperature of a fully engulfed fire in an office has to be determined. The room of the “Simulated Office” test of the Cardington building is chosen for this analysis. The measured temperatures during the fully engulfed fire are shown in Figure 3, so the calculation can be compared with these results.

A natural fire model is chosen for the calculation of the gas temperature. For fires with a flash-over, the method of the compartment fires can be used. A simple calculation method for a parametric temperature-time curve is given in Annex A of EN 1991-1-2.

Figure 1. Cardington building (left) and the office of the “Simulated Office” test (right)

Floor area: A_f = 135 m²
Total area of vertical openings: A_v = 27 m²
Vertical opening factor: α_v = 0.2
Horizontal opening factor: α_h = 0.0
Height: H = 4.0 m
Average window height: h_{eq} = 1.8 m (assumption)
Lightweight concrete: ρ = 1900 kg/m²
c = 840 J/kgK
λ = 1.0 W/mK

2 DETERMINATION OF FIRE LOAD DENSITY

EN 1991-1-2

For the determination of the fire load density the Annex E of EN 1991-1-2 offers a calculation model. The design value of the load density may either be given from a national fire load classification of occupancies and/or specific for an individual project by performing a fire load evaluation.

At this example, the second method is chosen.

Annex E.1

$$q_{f,d} = q_{f,k} \cdot m \cdot \delta_{q1} \cdot \delta_{q2} \cdot \delta_n$$

where:

m the combustion factor

δ_{q1} the factor considering the danger of fire activation by size of the compartment

δ_{q2} the factor considering the fire activation risk due to the type of occupancy

δ_n the factor considering the different active fire fighting measures

The fire load consisted of 20 % plastics, 11 % paper and 69 % wood, so it consisted mainly of cellulosic material. Therefore the combustion factor is:

$$m = 0.8$$

The factor δ_{q1} considers the danger of fire activation by size of the compartment, as given in Table 1.

Table 1. Fire activation risk due to the size of the compartment (see EN 1991-1-2, Table E.1)

	Compartment floor area A_f [m²]				
	≤ 25	≤ 250	≤ 2500	≤ 5000	≤ 10,000
Danger of fire activation δ_{q1}	1.10	1.50	1.90	2.00	2.13

$$\delta_{q1} = 1.5$$

A factor δ_{q2} considers the fire activation risk due to the type of occupancy, as given in Table 2.

Table 2. Fire activation risk due to the type of occupancy (see EN 1991-1-2, Table E.1)

Danger of fire activation δ_{q2}	Examples of occupancies
0.78	artgallery, museum, swimming pool
1.00	offices, residence, hotel, paper industry
1.22	manufactory for machinery & engines
1.44	chemical laboratory, painting workshop
1.66	manufactory for fireworks or paints

$$\delta_{q2} = 1.5$$

The factor taking the different active fire fighting measures into account is calculated to:

$$\delta_n = \prod_{i=1}^{10} \delta_{ni}$$

The factors δ_{ni} are given in Table 3.

Table 3. Factors δ_{ni} (see EN 1991-1-2, Table E.2)

δ_{ni} function of active fire fighting measures				
Automatic fire suppression	Automatic water extinguishing system	δ_{n1}		0.61
	Independent water supplies	δ_{n2}	0 1 2	1.0 0.87 0.7
Automatic fire detection	Automatic fire detection & alarm	δ_{n3}	By heat or	0.87
		δ_{n4}	by smoke	0.73
	Automatic alarm transmission to fire brigade	δ_{n5}		0.87
Manual fire suppression	Work Fire Brigade	δ_{n6}		0.61
	Off Site Fire Brigade	δ_{n7}		0.78
	Safe access routes	δ_{n8}		0.9 or 1.0 or 1.5
	Fire fighting devices	δ_{n9}		1.0 or 1.5
	Smoke exhaust system	δ_{n10}		1.0 or 1.5

$$\delta_n = 1.0 \cdot 0.73 \cdot 0.87 \cdot 0.78 \cdot 1.0 \cdot 1.0 \cdot 1.0 = 0.50$$

For calculating the characteristic fire load, the characteristic fire load has to be determined. It is defined as:

$$Q_{fi,k} = \sum M_{k,i} \cdot H_{ui} \cdot \psi_i$$

Annex E.2

where:

$M_{k,i}$ the amount of combustible material [kg]
H_{ui} the net calorific value [MJ/kg], see EN 1991-1-2, Table E.3
ψ_i the optional factor for assessing protected fire loads

The total fire loading was equivalent to 46 kg wood/m², so the characteristic fire load is:

$$Q_{fi,k} = (135 \cdot 46) \cdot 17.5 \cdot 1.0 = 108{,}675 \text{ MJ}$$

The characteristic fire load density is determined to:

$$q_{f,k} = Q_{fi,k} / A_f = 108{,}675/135 = 805 \text{ MJ/m}^2$$

The design value of the fire load density is calculated to:

$$\begin{aligned} q_{f,d} &= 805 \cdot 0.8 \cdot 1.5 \cdot 1.0 \cdot 0.5 \\ &= 483.0 \text{ MJ/m}^2 \end{aligned}$$

3 CALCULATION OF THE PARAMETRIC TEMPERATURE-TIME CURVE

It has to be determined if the fully engulfed fire is fuel or ventilation controlled. For this, the opening factor and the design value of the fire load density related to the total surface are needed.

$$O = \sqrt{h_{eq}} \cdot A_v / A_t = \sqrt{1.8} \cdot 27/474 = 0.076 \text{ m}^{1/2} \begin{cases} \geq 0.02 \\ \leq 0.2 \end{cases}$$

and

$$q_{t,d} = q_{f,d} \cdot A_f / A_t = 483.0 \cdot 135/474 = 137.6 \, \text{MJ/m}^2$$

The determination, if the fire is fuel or ventilation controlled is:

$$0.2 \cdot 10^{-3} \cdot q_{t,d} / O = 0.2 \cdot 10^{-3} \cdot 137.6/0.076 = 0.362 \text{ h} > t_{\text{lim}} = 0.333 \text{ h}$$

⇒ The fire is ventilation controlled

For calculation of the temperature-time curves for the heating and the cooling phase, the *b* factor is needed. This factor considers the thermal absorptivity for the boundary of enclosure. The density, the specific heat and the thermal conductivity of the boundary may be taken at ambient temperature. The floor, the slab and the walls are made of lightweight concrete

$$b = \sqrt{\rho \cdot c \cdot \lambda} = \sqrt{1900 \cdot 840 \cdot 1.0} = 1263.3 \frac{\text{J}}{\text{m}^2 \text{s}^{1/2} \text{K}} \quad \begin{cases} \geq 100 \\ \leq 2200 \end{cases}$$

The temperature-time curve in the heating phase is given by:

$$\theta_g = 20 + 1325 \cdot \left(1 - 0.324 \cdot \text{e}^{-0.2 \cdot t^*} - 0.204 \cdot \text{e}^{-1.7 \cdot t^*} - 0.472 \cdot \text{e}^{-19 \cdot t^*}\right)$$

Because the fire is ventilation controlled, the time *t** is calculated to:

$$t^* = t \cdot \Gamma$$

where:

$$\Gamma = \frac{(O/b)^2}{(0.04/1160)^2} = \frac{(0.076/1263.3)^2}{(0.04/1160)^2} = 3.04$$

Now the heating phase can be calculated:

$$\theta_g = 20 + 1325 \cdot \left(1 - 0.324 \cdot \text{e}^{-0.2 \cdot (3.04 \cdot t)} - 0.204 \cdot \text{e}^{-1.7 \cdot (3.04 \cdot t)} - 0.472 \cdot \text{e}^{-19 \cdot (3.04 \cdot t)}\right)$$

For calculation of the cooling phase, the maximum temperature is needed.

$$\theta_{\max} = 20 + 1325 \cdot \left(1 - 0.324 \cdot e^{-0.2 \cdot t^*_{\max}} - 0.204 \cdot e^{-1.7 \cdot t^*_{\max}} - 0.427 \cdot e^{-19 \cdot t^*_{\max}}\right)$$

where:

$$t^*_{\max} = t_{\max} \cdot \Gamma$$

The time $t_{\max}$ is determined as below, where t_{lim} is given in Table 4.

$$t_{\max} = \max \begin{cases} 0.2 \cdot 10^{-3} \cdot q_{t,d} / O = 0.2 \cdot 10^{-3} \cdot 137.6/0.076 = 0.362 \text{ h} \\ t_{\text{lim}} = 0.333 \text{ h} \end{cases}$$

Table 4. Time t_{lim} for different fire growth rates

	Slow fire growth rate	Medium fire growth rate	Fast fire growth rate
t_{lim} [h]	0.417	0.333	0.250

So $t^*_{\max}$ is calculated to:

$$t^*_{\max} = 0.362 \cdot 3.04 = 1.10 \text{ h}$$

The maximum temperature is calculated to:

$$\theta_{\max} = 20 + 1325 \cdot \left(1 - 0.324 \cdot e^{-0.2 \cdot 1.10} - 0.204 \cdot e^{-1.7 \cdot 1.10} - 0.427 \cdot e^{-19 \cdot 1.10}\right)$$
$$= 958.8 \text{ °C}$$

During the cooling phase, t^* and $t^*_{\max}$ are calculated to:

$$t^* = t \cdot \Gamma = t \cdot 3.04 \quad [\text{h}]$$

$$t^*_{\max} = \left(0.2 \cdot 10^{-3} \cdot q_{t,d} / O\right) \cdot \Gamma = 1.10 \text{h}$$

The temperature-time curve in the cooling phase for $t^*_{\max} \leq 0.5$ is given by:

$$\theta_g = \theta_{\max} - 625 \cdot \left(t^* - t^*_{\max} \cdot x\right)$$
$$= 958.8 - 625 \cdot \left(t \cdot 3.04 - 1.10 \cdot 1.0\right)$$

where:

$t_{\max} > t_{\text{lim}}$ $\quad x = 1.0$

Combination of the heating and cooling curves leads to the parametric temperature-time curve shown in Figure 2.

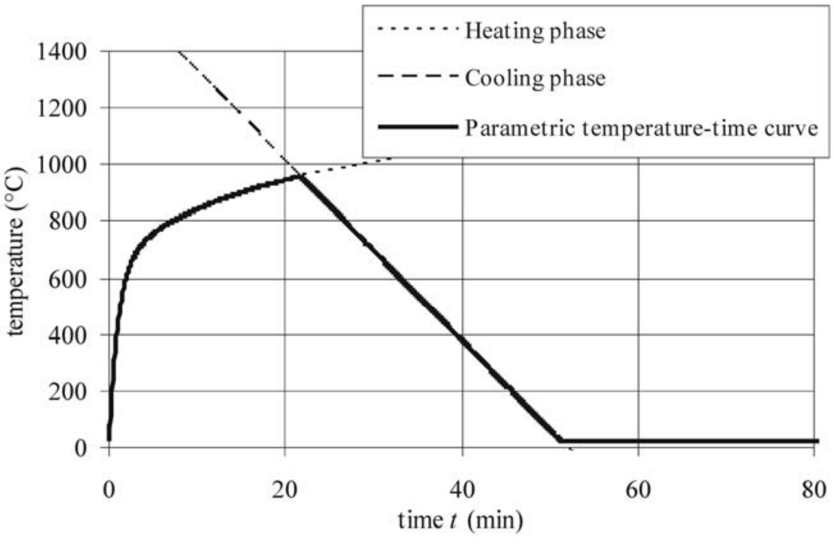


Figure 2. Gas temperature of the office calculated by using the parametric temperature-time curve of the office

4 COMPARISON BETWEEN CALCULATION AND FIRE TEST

To compare the calculation with the measured temperatures in the test, the factors δ_1, δ_2 and δ_{ni} for calculation of the fire load density have to be set to 1.0 (see Figure 3).

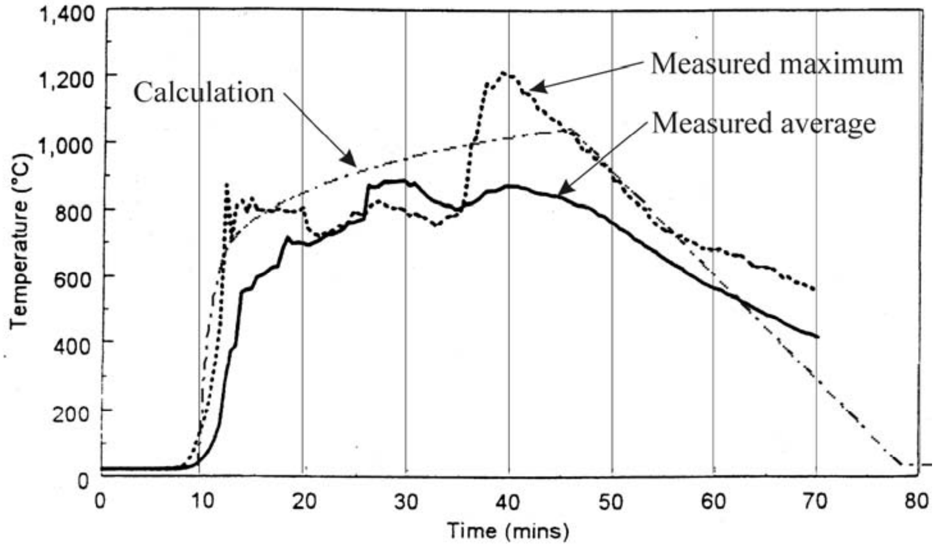


Figure 3. Comparison of measured and calculated temperature-time curves

REFERENCES

EN 1991, *Eurocode 1:Actions on structures – Part 1-2: General actions – Actions on structures exposed to fire*, Brussels: CEN, November 2002

The Behaviour of multi-storey steel framed buildings in fire, Moorgate: British Steel plc, Swinden Technology Centre, 1998

Valorisation Project: Natural Fire Safety Concept, Sponsored by ECSC, June 2001

Example to EN 1991 Part 1-2: Localised fire

P. Schaumann, T. Trautmann
University of Hannover – Institute for Steel Construction, Hannover, Germany

1 TASK

The steel temperature of a beam has to be determined. It is part of an underground car park below the shopping mall Auchan in Luxembourg. The beams of the car park are accomplished without any use of fire protection material. The most severe fire scenario is a burning car in the middle of the beam (see Figure 1).

For getting the steel temperature, the natural fire model of a localised fire is used.

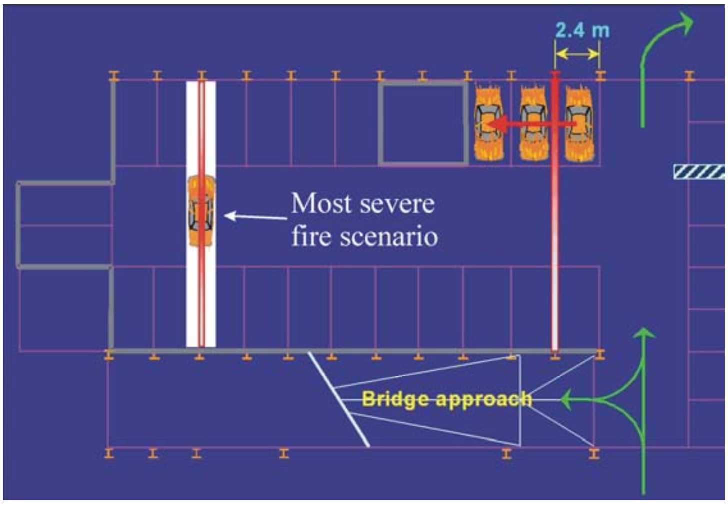


Figure 1. Underground car park of the shopping mall Auchan

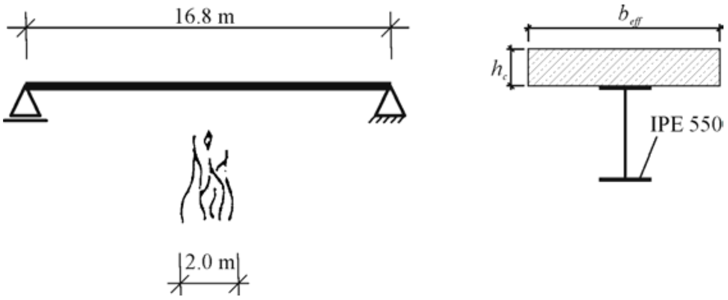


Figure 2. Static system and cross-section of the beam

Diameter of the fire: $D = 2.0$ m
Vertical distance between fire source and ceiling: $H = 2.7$ m
Horizontal distance between beam and flame axis: $r = 0.0$ m
Emissivity of the fire: $\varepsilon_f = 1.0$
Configuration factor: $\Phi = 1.0$
Stephan Boltzmann constant: $\sigma = 5.56 \cdot 10^{-8}$ W/m^{2}K^4
Coefficient of the heat transfer: $\alpha_c = 25.0$ W/m^{2}K

Steel profile: IPE 550
Section factor: $A_m/V = 140$ 1/m
Unit mass: $\rho_a = 7850$ kg/m^3
Surface emissivity: $\varepsilon_m = 0.7$
Correction factor: $k_{sh} = 1.0$

2 RATE OF HEAT RELEASE

ECSC Project

The rate of heat release is normally determined by using the EN 1991-1-2 Section E.4. For dimensioning the beams at this car park, the rate of heat release for one car is taken from an ECSC project called "Development of design rules for steel structures subjected to natural fires in CLOSED CAR PARKS" (see Figure 3).

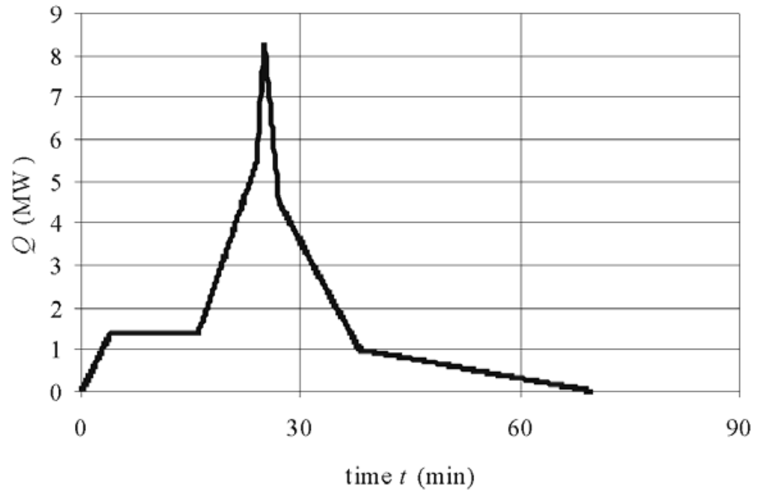


Figure 3. Rate of heat release of one car

3 CALCULATION OF THE STEEL TEMPERATURES

EN 1991-1-2

3.1 *Calculation of the flame length*

Annex C

First of all, the flame length has to be determined.

$$L_f = -1.02 \cdot D + 0.0148 \cdot Q^{2/5} = -2.04 + 0.0148 \cdot Q^{2/5}$$

A plot of this function with the values of Figure 3 is shown in Figure 4. With a ceiling height of 2.80 m, the flame is impacting the ceiling at a time from 16.9 min to 35.3 min (see Figure 4).

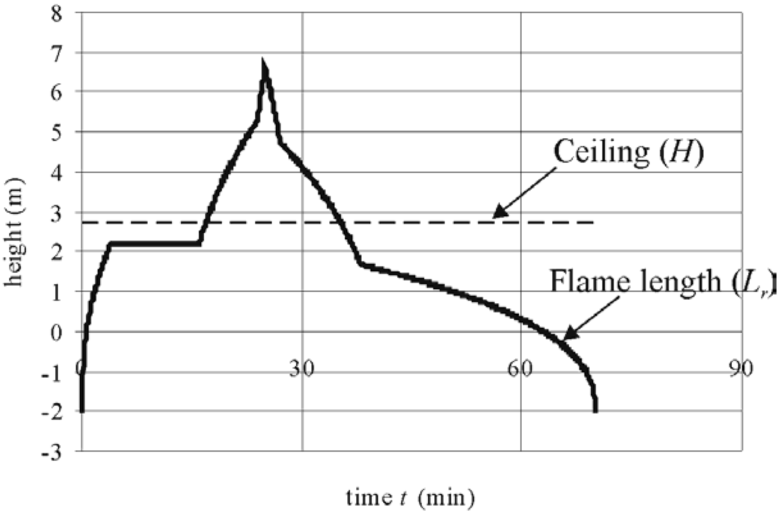


Figure 4. Flame length of the localised fire

It is important to know, if the flame is impacting the ceiling or not, because different calculation methods for the calculation of the net heat flux are used for these two cases (see Figure 5).

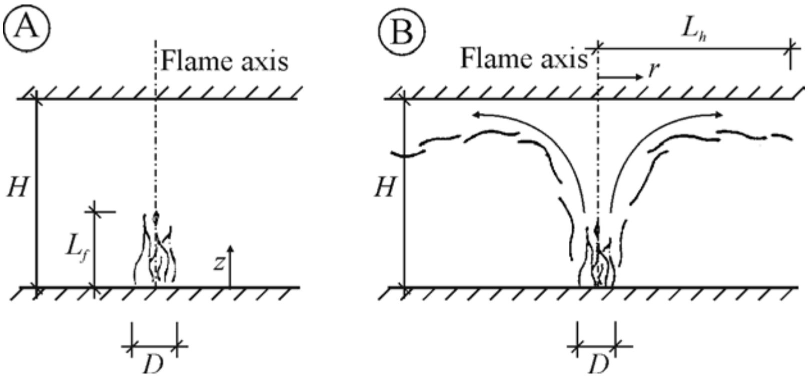


Figure 5. Flame models: Flame is not impacting the ceiling (A); Flame is impacting the ceiling (B)

3.2 *Calculation of the net heat flux*

3.2.1 *1^{st} case: The flame is not impacting the ceiling*

The net heat flux is calculated according to Section 3.1 of EN 1991-1-2.

Section 3.1

$$\dot{h}_{net} = \alpha_c \cdot \left(\theta_{(z)} - \theta_m\right) + \Phi \cdot \varepsilon_m \cdot \varepsilon_f \cdot \sigma \cdot \left(\left(\theta_{(z)} + 273\right)^4 - \left(\theta_m + 273\right)^4\right)$$

$$= 25.0 \cdot \left(\theta_{(z)} - \theta_m\right) + 3.892 \cdot 10^{-8} \cdot \left(\left(\theta_{(z)} + 273\right)^4 - \left(\theta_m + 273\right)^4\right)$$

The gas temperature is calculated to:

$$\theta_{(z)} = 20 + 0.25 \cdot \left(0.8 \cdot Q\right)^{2/3} \cdot \left(z - z_0\right)^{-5/3} \leq 900\ °\mathrm{C}$$

$$= 20 + 0.25 \cdot \left(0.8 \cdot Q\right)^{2/3} \cdot \left(0.66 - 0.0052 \cdot Q^{2/5}\right)^{-5/3} \leq 900\ °\mathrm{C}$$

where:

z is the height along the flame axis (2.7 m)

z_0 is the virtual origin of the axis [m]

$$z_0 = -1.02 \cdot D + 0.0052 \cdot Q^{2/5} = -2.04 + 0.0052 \cdot Q^{2/5}$$

3.2.2 2

The net heat flux, if the flame is impacting the ceiling, is given by:

$$\dot{h}_{net} = \dot{h} - \alpha_c \cdot \left(\theta_m - 20\right) - \Phi \cdot \varepsilon_m \cdot \varepsilon_f \cdot \sigma \cdot \left(\left(\theta_m + 273\right)^4 - \left(293\right)^4\right)$$

$$= \dot{h} - 25.0 \cdot \left(\theta_m - 20\right) - 3.892 \cdot 10^{-8} \cdot \left(\left(\theta_m + 273\right)^4 - \left(293\right)^4\right)$$

The heat flux depends on the parameter *y*. For different dimensions of *y*, different equations for determination of the heat flux have to be used.

if $y \leq 0.30$:

$$\dot{h} = 100{,}000$$

if $0.30 < y < 1.0$:

$$\dot{h} = 136{,}300 - 121{,}000 \cdot y$$

if $y \geq 1.0$:

$$\dot{h} = 15{,}000 \cdot y^{-3.7}$$

where:

$$y = \frac{r + H + z'}{L_h + H + z'} = \frac{2.7 + z'}{L_h + 2.7 + z'}$$

The horizontal flame length is calculated to:

$$L_h = \left(2.9 \cdot H \cdot \left(Q_H^{*}\right)^{0.33}\right) - H = \left(7.83 \cdot \left(Q_H^{*}\right)^{0.33}\right) - 2.7$$

where:

$$Q_H^{*} = Q \big/ \left(1.11 \cdot 10^6 \cdot H^{2.5}\right) = Q \big/ \left(1.11 \cdot 10^6 \cdot 2.7^{2.5}\right)$$

The vertical position of the virtual heat source is determined to:

if $Q_D^{*} < 1.0$:

$$z' = 2.4 \cdot D \cdot \left(\left(Q_D^{*}\right)^{2/5} - \left(Q_D^{*}\right)^{2/3}\right) = 4.8 \cdot \left(\left(Q_D^{*}\right)^{2/5} - \left(Q_D^{*}\right)^{2/3}\right)$$

if $Q_D^{*} \geq 1.0$:

$$z' = 2.4 \cdot D \cdot \left(1.0 - \left(Q_D^{*}\right)^{2/5}\right) = 4.8 \cdot \left(1.0 - \left(Q_D^{*}\right)^{2/5}\right)$$

where:

$$Q_D^{*} = Q \big/ \left(1.11 \cdot 10^6 \cdot D^{2.5}\right) = Q \big/ \left(1.11 \cdot 10^6 \cdot 2.0^{2.5}\right)$$

3.3 *Calculation of the steel temperature-time curve*

EN 1993-1-2

The specific heat of the steel c_a is needed to calculate the steel temperature. The parameter is given by EN 1993-1-2, Section 3.4.1.2 depending on the steel temperature.

Section 3.4.1.2

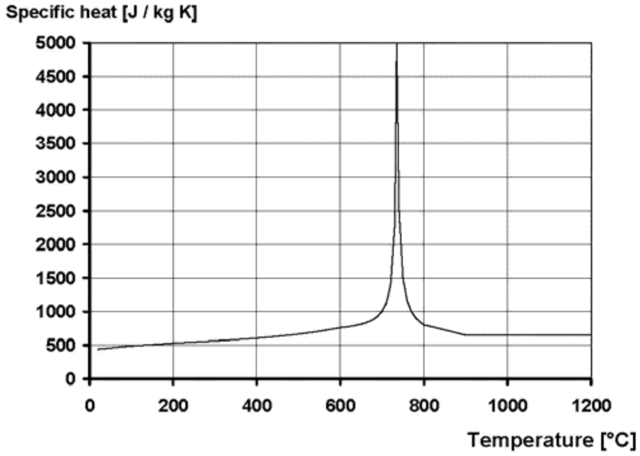


Figure 6. Specific heat of carbon steel (see EN 1993 Part 1-2, Figure 3.4)

$$\theta_{a,t} = \theta_m + k_{sh} \cdot \frac{A_m/V}{c_a \cdot \rho_a} \cdot \dot{h}_{net} \cdot \Delta t = \theta_m + 1.49 \cdot 10^{-4} \cdot \dot{h}_{net}$$

Section 4.2.5.1

The steel temperature-time curve is shown in Figure 6. Additionally, the results of the FEM-analysis done by PROFILARBED are shown for comparison.

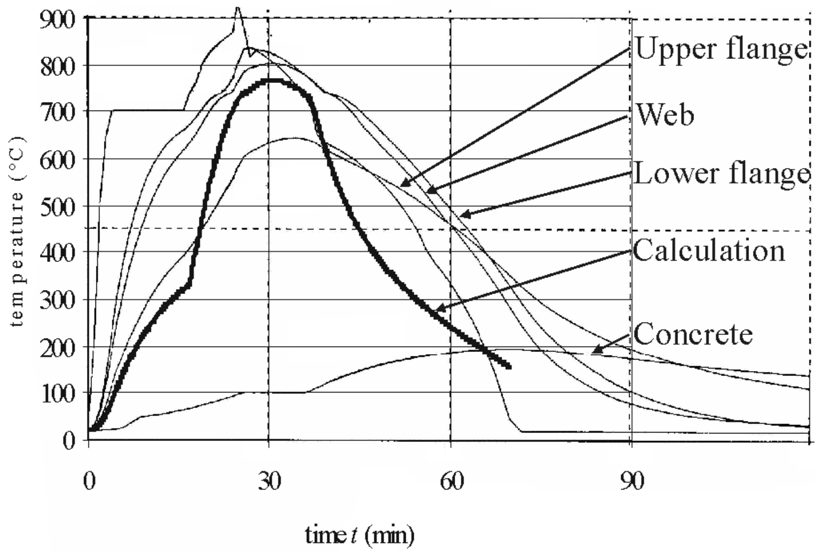


Figure 7. Comparison of the temperature-time curve of the calculation and the FEM-analysis of PROFILARBED

REFERENCES

EN 1991, *Eurocode 1:Actions on structures – Part 1-2: General actions – Actions on structures exposed to fire*, Brussels: CEN, November 2002

EN 1993, *Eurocode 3: Design of steel structures – Part 1-2: General rules – Structural fire design*, Brussels: CEN

ECSC Project, *Development of design rules for steel structures subjected to natural fires in CLOSED CAR PARKS*, CEC agreement 7210-SA/211/318/518/620/933, Brussels, June 1996

Example to EN 1993 Part 1-2: Column with axial loads

P. Schaumann, T. Trautmann
University of Hannover – Institute for Steel Construction, Hannover, Germany

1 TASK

In the following example, a column of a department store will be dimensioned for fire resistance. The column is part of a braced frame and is connected bending resistant to the upper and lower column. The length is 3.0 m. During fire exposure, the buckling length can be reduced as seen below in Figure 1. The loads are centric axial compression forces. The column is exposed to fire on four sides. A hollow encasement of gypsum is chosen for fire protection. The required standard fire resistance class for the column is R 90.

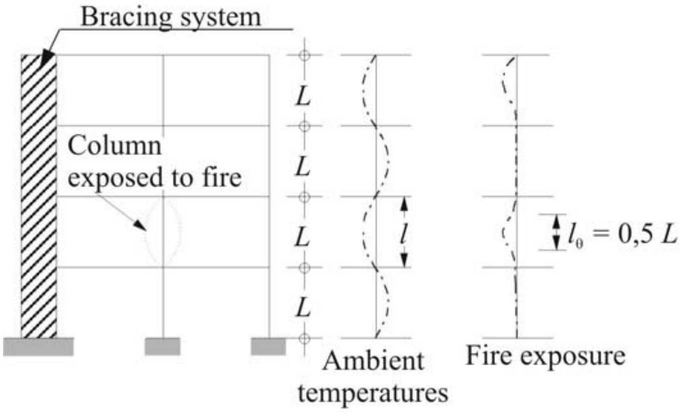


Figure 1. Buckling lengths of columns in braced frames

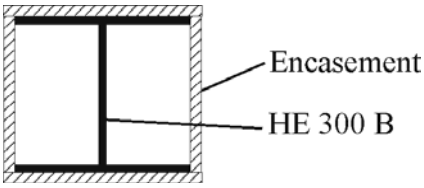


Figure 2. Cross-section of the column

Material properties:
- Column:
  - Profile: rolled section HE 300 B
  - Steel grade: S 235
  - Cross-section class: 1
  - Yield stress: f_y = 23.5 kN/cm²
  - Cross-sectional area: A_a = 149 cm²

Elastic modulus: E_a = 21,000 kN/cm²
Moment of inertia: I_a = 8560 cm^4 (weak axis)
Encasement:
Material: gypsum
Thickness: d_p = 3.0 cm (hollow encasement)
Thermal conductivity: λ_p = 0.2 W/(m·K)
Specific heat: c_p = 1700 J/(kg·K)
Density: ρ_p = 945 kg/m³
Loads:
Permanent actions: G_k = 1200 kN
Variable actions: P_k = 600 kN

2 FIRE RESISTANCE OF COLUMN

2.1 *Mechanical actions during fire exposure*

EN 1991-1-2

The accidental situation is used for the combination of mechanical actions during fire exposure.

$$E_{dA} = E\left(\sum G_k + A_d + \sum \psi_{2,i} \cdot Q_{k,i}\right)$$

Section 4.3

The combination factor for department stores is $\psi_{2,1}$ = 0.6. So the axial load is determined to:

$$N_{fi,d} = 1200 + 0.6 \cdot 600 = 1560 \text{ kN}$$

2.2 *Calculation of the maximum steel temperature*

EN 1993-1-2

The analysis of EN 1993-1-2 is used to calculate the steel temperature of the hollow encased column. For a hollow encased member, the section factor is calculated to:

$$A_p/V = 2 \cdot (b+h)/A_a = 2 \cdot (30+30) \cdot 10^2/149 = 81 \text{ m}^{-1}$$

Section 4.2.5.2

By using the Euro-Nomogram (ECCS No.89), the maximal temperature $\theta_{a,max,90}$ of the steel bar is:

$$\left(A_p/V\right) \cdot \left(\lambda_p/d_p\right) = 81 \cdot 0.2/0.03 = 540 \text{ W/m}^3\text{K}$$

ECCS No.89

$$\Rightarrow \ \theta_{a,max,90} \approx 445\ °\text{C}$$

2.3 *Verification in the temperature domain*

EN 1993-1-2

Within EN 1993-1-2 the verification in the temperature domain is not allowed for members in which stability phenomena have to be taken into account.

Section 4.2.4

2.4 *Verification in the strength domain*

The verification in the strength domain during fire exposure is carried out as a plastic ultimate state of the load-carrying capacity.

$$E_{fi,d,t} \le R_{fi,d,t}$$

Section 2.4.2

In this example, the verification has to be done with the axial forces.

$$N_{fi,d} \le N_{b,fi,t,Rd}$$

The design resistance under high temperature conditions is calculated as:

$$N_{b,fi,t,Rd} = \chi_{fi} \cdot A_a \cdot k_{y,\theta,\max} \cdot \frac{f_y}{\gamma_{M,fi}}$$

Section 4.2.3.2

In dependence of $\theta_{a,max,90}$ the reduction factors $k_{y,\theta}$ and $k_{E,\theta}$ are given in Table 3.1 of the EN 1993-1-2. For intermediate values of the steel temperature, linear interpolation may be used.

$$\Rightarrow \quad k_{y,445°C} = 0.901$$

Section 3.2.1

$$k_{E,445°C} = 0.655$$

The load-carrying capacity is determined in consideration of the non-dimensional slenderness during fire exposure.

$$\bar{\lambda}_{fi,\theta} = \bar{\lambda} \cdot \sqrt{k_{y,\theta}/k_{E,\theta}} = 0.21 \cdot \sqrt{0.901/0.655} = 0.25$$

Section 4.2.3.2

where:

EN 1993-1-1

$$\bar{\lambda} = L_{Kz}/\left(i_z \cdot \lambda_a\right) = \left(0.5 \cdot 300\right)/\left(7.58 \cdot 93.9\right) = 0.21$$

Section 6.3.1.3

With the non-dimensional slenderness the reduction factor for flexural buckling $\chi_{fi,\theta}$ can be calculated.

EN 1993-1-2

$$\chi_{fi} = \frac{1}{\varphi + \sqrt{\varphi^2 - \bar{\lambda}^2}} = \frac{1}{0.61 + \sqrt{0.61^2 - 0.14^2}} = 0.86$$

Section 4.2.3.2

where:

$$\varphi \ = \ 0.5 \cdot \left[1 + \alpha \cdot \bar{\lambda} + \bar{\lambda}^2\right] = \ 0.5 \cdot \left[1 + 0.65 \cdot 0.25 + 0.25^2\right] = 0.61$$

and:

$$\alpha = 0.65 \cdot \sqrt{235/f_y} = 0.65 \cdot \sqrt{235/235} = 0.65$$

The design resistance arises to:

$$N_{b,fi,t,Rd} = 0.86 \cdot 149 \cdot 0.901 \cdot \frac{23.5}{1.0} = 2713 \text{ kN}$$

Verification:

$$N_{fi,d}/N_{b,fi,t,Rd} = 1560/2713 = 0.58 < 1 \qquad \checkmark$$

REFERENCES

ECCS No.89, *Euro-Nomogram*, Brussels: ECCS – Technical Committee 3 – Fire Safety of Steel Structures, 1995

EN 1991, *Eurocode 1:Actions on structures – Part 1-2: General actions – Actions on structures exposed to fire*, Brussels: CEN, November 2002

EN 1993, *Eurocode 3: Design of steel structures – Part 1-1: General rules*, Brussels: CEN, May 2002

EN 1993, *Eurocode 3: Design of steel structures – Part 1-2: General rules – Structural fire design*, Brussels: CEN, November 2003

Example to EN 1993 Part 1-2: Beam with bending and compression loads

P. Schaumann, T. Trautmann
University of Hannover – Institute for Steel Construction, Hannover, Germany

1 TASK

This example deals with a beam subjected to a uniform load, which causes bending moment, and an axial load. Stability phenomena have to be considered. The beam is part of an office building. A hollow encasement of gypsum is chosen for fire protection. Due to a concrete slab the beam is exposed to fire on three sides. There is no shear-connection between the beam and the slab. The required standard fire resistance class for the beam is R 90.

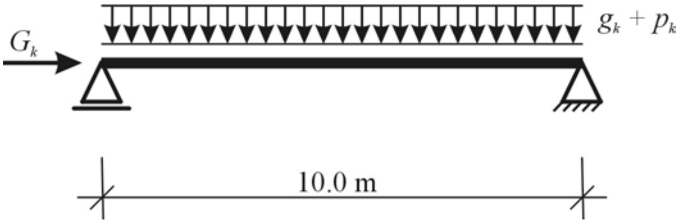


Figure 1. Static system

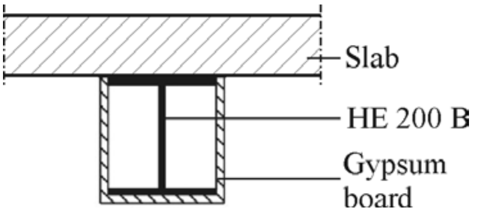


Figure 2. Beam Cross-section

Material properties:

Beam:

Profile: rolled section HE 200 B
Steel grade: S 235
Cross-section class: 1
Yield stress: $f_y = 235$ N/mm²
Elastic modulus: $E = 210{,}000$ N/mm²
Shear modulus: $G = 81{,}000$ N/mm²
Cross-sectional area: $A_a = 7810$ mm²
Moment of inertia: $I_z = 2000$ cm^4
Torsion constant: $I_t = 59.3$ cm^4

Warping constant: I_w = 171,100 cm^6
Section moduli: $W_{el,y}$ = 570 cm^2
$W_{pl,y}$ = 642.5 cm^3

Encasement:
Material: gypsum
Thickness: d_p = 20 mm (hollow encasement)
Thermal conductivity: λ_p = 0.2 W/(m·K)
Specific heat: c_p = 1700 J/(kg·K)
Density: ρ_p = 945 kg/m^3

Loads:
Permanent Loads: G_k = 96.3 kN
g_k = 1.5 kN/m
Variable Loads: p_k = 1.5 kN/m

2 FIRE RESISTANCE OF BEAM WITH BENDING AND COMPRESSION LOADS

2.1 *Mechanical actions during fire exposure*

EN 1991-1-2

The combination of mechanical actions during fire exposure shall be calculated as an accidental situation:

$$E_{dA} = E\left(\sum G_k + A_d + \sum \psi_{2,i} \cdot Q_{k,i}\right)$$

Section 4.3

The combination factor for office buildings is $\psi_{2,1} = 0.3$. The design loads under high temperature conditions are:

$$N_{fi,d} = 96.3 \text{ kN}$$

$$M_{fi,d} = \left[1.5 + 0.3 \cdot 1.5\right] \cdot \frac{10.0^2}{8} = 24.38 \text{ kNm}$$

2.2 *Calculation of steel temperatures*

prEN 1993-1-2

The steel temperature is given by the Euro-Nomogram (ECCS No.89). Therefore the section factor A_p/V is needed. For a hollow encased member exposed to fire on three sides, the section factor is:

$$\frac{A_p}{V} = \frac{2 \cdot h + b}{A_a} = \frac{2 \cdot 20.0 + 20.0}{78.1} \cdot 10^2 = 77 \text{ m}^{-1}$$

Section 4.2.5.2

With

$$\frac{A_p}{V} \cdot \frac{\lambda_p}{d_p} = 77 \cdot \frac{0.2}{0.02} = 770 \ \frac{\text{W}}{\text{m}^3 \cdot \text{K}},$$

ECCS No.89

the critical temperature arises to:

$$\Rightarrow \quad \theta_{a,max,90} \approx 540 \text{ °C}$$

2.3 *Verification in the temperature domain*

prEN 1993-1-2

Due to section 4.2.4 (2) of prEN 1993-1-2 the verification in the temperature domain may not be accomplished, because of stability problems of the beam.

Section 4.2.4

2.4 *Verification in the strength domain*

Members with a Class 1 cross-section should be analysed for the problem of flexural buckling and of lateral torsional buckling.

2.4.1 *Flexural buckling*

The verification for flexural buckling is:

$$\frac{N_{fi,d}}{\chi_{\min,fi} \cdot A \cdot k_{y,\theta} \cdot f_y / \gamma_{M,fi}} + \frac{k_y \cdot M_{y,fi,d}}{W_{pl,y} \cdot k_{y,\theta} \cdot f_y / \gamma_{M,fi}} \leq 1$$

Section 4.2.3.5

The reduction factor $\chi_{min,fi}$ is the minimum of the two reduction factors for flexural buckling $\chi_{y,fi}$ and $\chi_{z,fi}$. The non-dimensional slenderness for the temperature θ_a is needed for the calculation of these reduction factors.

For calculation of the non-dimensional slenderness in the fire situation, the non-dimensional slenderness at ambient temperatures have to be determined.

prEN 1993-1-1

$$\bar{\lambda}_y = \frac{L_{cr}}{i_y \cdot \lambda_a} = \frac{1000}{8.54 \cdot 93.9} = 1.25$$

Section 6.3.1.3

$$\bar{\lambda}_z = \frac{L_{cr}}{i_z \cdot \lambda_a} = \frac{1000}{5.07 \cdot 93.9} = 2.10$$

The needed reduction factors $k_{y,\theta}$ and $k_{E,\theta}$ are given in prEN 1993-1-2 Table 3.1:

prEN 1993-1-2

$$\Rightarrow \qquad k_{y,\theta} = 0.656$$

Section 3.2.1

$$k_{E,\theta} = 0.484$$

With the reduction factors, the non-dimensional slenderness in the fire situation can be determined:

$$\bar{\lambda}_{y,\theta} = \bar{\lambda}_y \sqrt{\frac{k_{y,\theta}}{k_{E,\theta}}} = 1.25\sqrt{\frac{0.656}{0.484}} = 1.46$$

Section 4.2.3.2

$$\bar{\lambda}_{z,\theta} = \bar{\lambda}_z \sqrt{\frac{k_{y,\theta}}{k_{E,\theta}}} = 2.1\sqrt{\frac{0.656}{0.484}} = 2.44$$

With

$$\alpha = 0.65 \cdot \sqrt{235/f_y} = 0.65 \cdot \sqrt{235/235} = 0.65$$

and

$$\varphi_{y,\theta} = \frac{1}{2} \cdot \left(1 + \alpha \cdot \bar{\lambda}_{y,\theta} + \bar{\lambda}_{y,\theta}^{\,2}\right) = \frac{1}{2} \cdot \left(1 + 0.65 \cdot 1.46 + 1.46^2\right) = 2.04\,,$$

$$\varphi_{z,\theta} = \frac{1}{2} \cdot \left(1 + \alpha \cdot \bar{\lambda}_{z,\theta} + \bar{\lambda}_{z,\theta}^{\,2}\right) = \frac{1}{2} \cdot \left(1 + 0.65 \cdot 2.44 + 2.44^2\right) = 4.27$$

the reduction factors $\chi_{y,fi}$ and $\chi_{z,fi}$ can be calculated:

$$\chi_{y,fi} = \frac{1}{\varphi_{y,\theta} + \sqrt{\varphi_{y,\theta}^{\;2} - \bar{\lambda}_{y,\theta}^{\;2}}} = \frac{1}{2.04 + \sqrt{2.04^2 - 1.46^2}} = 0.29$$

$$\chi_{z,fi} = \frac{1}{\varphi_{z,\theta} + \sqrt{\varphi_{z,\theta}^{\;2} - \bar{\lambda}_{z,\theta}^{\;2}}} = \frac{1}{4.27 + \sqrt{4.27^2 - 2.44^2}} = 0.13$$

Verification:

$$\frac{96.3}{0.13 \cdot 78.1 \cdot 0.656 \cdot 23.5} + \frac{1.33 \cdot 2438}{642.5 \cdot 0.656 \cdot 23.5} = 0.94 < 1 \qquad \checkmark$$

Section 4.2.3.5

where:

$$\begin{aligned} \mu_y &= \left(1.2 \cdot \beta_{M,y} - 3\right) \cdot \bar{\lambda}_{y,\theta} + 0.44 \cdot \beta_{M,y} - 0.29 \\ &= \left(1.2 \cdot 1.3 - 3\right) \cdot 1.46 + 0.44 \cdot 1.3 - 0.29 \\ &= -1.82 \end{aligned}$$

$$k_y = 1 - \frac{\mu_y \cdot N_{fi,d}}{\chi_{y,fi} \cdot A_a \cdot f_y / \gamma_{m,fi}} = 1 - \frac{-1.82 \cdot 96.3}{0.29 \cdot 78.1 \cdot 23.5/1.0} = 1.33$$

2.4.2 *Lateral torsional buckling*

The second verification deals with the problem of lateral torsional buckling.

$$\frac{N_{fi,d}}{\chi_{z,fi} \cdot A \cdot k_{y,\theta} \cdot f_y / \gamma_{M,fi}} + \frac{k_{LT} \cdot M_{y,fi,d}}{\chi_{LT,fi} \cdot W_{pl,y} \cdot k_{y,\theta} \cdot f_y / \gamma_{M,fi}} \le 1$$

For calculation of the non-dimensional slenderness in the fire situation, the non-dimensional slenderness at ambient temperatures has to be determined

prEN 1993-1-1

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_{pl,y} \cdot f_y}{M_{cr}}} = \sqrt{\frac{642.5 \cdot 23.5}{14{,}420.4}} = 1.05$$

Section 6.3.2.2

where:

$$M_{cr} = C_1 \cdot \frac{\pi^2 \cdot E \cdot I_z}{\left(k \cdot L\right)^2} \cdot \left[\sqrt{\left(\frac{k}{k_w}\right)^2 \frac{I_w}{I_z} + \frac{\left(k \cdot L\right)^2 \cdot G \cdot I_t}{\pi^2 \cdot E \cdot I_z} + \left(C_2 \cdot z_g\right)^2} - C_2 \cdot z_g \right]$$

Section C.2.2

$$= 1.12 \cdot \frac{\pi^2 \cdot 21{,}000 \cdot 2000}{\left(1.0 \cdot 1000\right)^2} \cdot$$

$$\left[\sqrt{\left(\frac{1.0}{1.0}\right)^2 \frac{171{,}100}{2000} + \frac{\left(1.0 \cdot 1000\right)^2 \cdot 8100 \cdot 59.3}{\pi^2 \cdot 21{,}000 \cdot 2000} + \left(0.45 \cdot \frac{20}{2}\right)^2} - 0.45 \cdot \frac{20}{2} \right]$$

$$= 14{,}420.4 \text{ kNcm}$$

During fire exposure, the non-dimensional slenderness changes to:

prEN 1993-1-2

$$\bar{\lambda}_{LT,\theta} = \bar{\lambda}_{LT} \cdot \sqrt{\frac{k_{y,\theta}}{k_{E,\theta}}} = 1.02 \cdot \sqrt{\frac{0.656}{0.484}} = 1.19$$

Section 4.2.3.3

With

$$\phi_{LT,\theta} = \frac{1}{2} \cdot \left(1 + \alpha \cdot \bar{\lambda}_{LT,\theta} + \bar{\lambda}_{LT,\theta}^{\ 2}\right) = \frac{1}{2} \cdot \left(1 + 0.65 \cdot 1.19 + 1.19^2\right) = 1.59\,,$$

the reduction factor $\chi_{LT,fi}$ is calculated to:

$$\chi_{LT,fi} = \frac{1}{\phi_{LT,\theta} + \sqrt{\phi_{LT,\theta}^{\ 2} - \bar{\lambda}_{LT,\theta}^{\ 2}}} = \frac{1}{1.59 + \sqrt{1.59^2 - 1.19^2}} = 0.38$$

Verification:

Section 4.2.3.5

$$\frac{96.3}{0.13 \cdot 78.1 \cdot 0.656 \cdot 23.5/1.0} + \frac{0.20 \cdot 2438}{0.38 \cdot 642.5 \cdot 0.656 \cdot 23.5/1.0}$$

$$= 0.60 + 0.13 = 0.73 \leq 1 \qquad \checkmark$$

where:

$$k_{LT} = \frac{\mu_{LT} \cdot N_{fi,d}}{\chi_{z,fi} \cdot A \cdot k_{y,\theta} \cdot f_y / \gamma_{M,fi}} = \frac{0.33 \cdot 93.3}{0.13 \cdot 78.1 \cdot 0.656 \cdot 23.5/1.0} = 0.20$$

$$\begin{aligned} \mu_{LT} &= 0.15 \cdot \bar{\lambda}_{z,\theta} \cdot \beta_{M,LT} - 0.15 < 0.9 \\ &= 0.15 \cdot 2.44 \cdot 1.3 - 0.15 \\ &= 0.33 < 0.9 \end{aligned}$$

REFERENCES

ECCS No.89, *Euro-Nomogram*, Brussels: ECCS – Technical Committee 3 – Fire Safety of Steel Structures, 1995

EN 1991, *Eurocode 1:Actions on structures – Part 1-2: General actions – Actions on structures exposed to fire*, Brussels: CEN

prEN 1993, *Eurocode 3: Design of steel structures – Part 1-1: General rules*, Brussels: CEN, May 2002

prEN 1993, *Eurocode 3: Design of steel structures – Part 1-2: General rules – Structural fire design*, Brussels: CEN

Example to EN 1993 Part 1-2: Beam made of a hollow section

P. Schaumann, T. Trautmann
University of Hannover – Institute for Steel Construction, Hannover, Germany

1 TASK

At this example, a beam made of a welded hollow section has to be dimensioned. It is part of a hall roof structure. The length of the beam is 35.0 m and the beams are arranged at a distance of 10.0 m. It is charged with uniform loads and is restrained against lateral evasion. The beam is executed without any use of fire protection material. The required standard fire resistance class for the tensile bar is R 30.

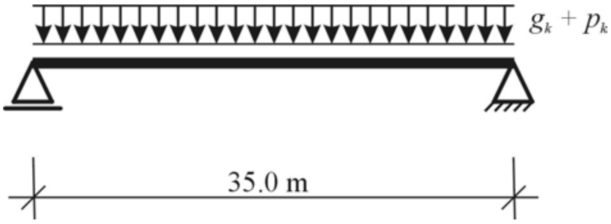


Figure 1. Static system

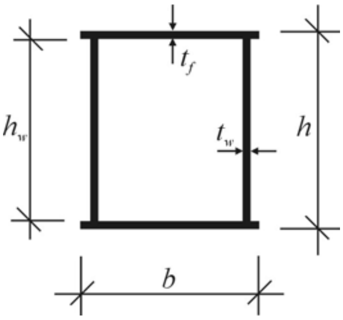


Figure 2. Cross-section

Material properties:

Steel grade: S 355
Yield stress: f_y = 355 N/mm²
Height: h = 700 mm
Height of web: h_w = 650 mm
Width: b = 450 mm
Thickness of flange: t_f = 25 mm
Thickness of web: t_w = 25 mm
Cross-sectional area of the flange: A_f = 11,250 mm²

Cross-sectional area of the web: A_w = 16,250 mm²
Specific heat: c_a = 600 J/(kg·K)
Density: ρ_a = 7850 kg/m³
Emissivity of the beam: ε_m = 0.7
Emissivity of the fire: ε_r = 1.0
Configuration factor Φ = 1.0
Coefficient of the heat transfer: α_c = 25.0 W/m²K
Stephan Boltzmann constant: $\sigma = 5.67 \cdot 10^{-8}$ W/m²K^4
Loads:
Permanent actions:
Beam: $g_{a,k}$ = 4.32 kN/m
Roof: $g_{r,k}$ = 5.0 kN/m
Variable actions:
Snow: $p_{s,k}$ = 11.25 kN/m

2 FIRE RESISTANCE OF BEAM MADE OF A HOLLOW SECTION

2.1 *Mechanical actions during fire exposure*

EN 1991-1-2

The accidental situation is used for the combination of mechanical actions during fire exposure.

$$E_{dA} = E\left(\sum G_k + A_d + \sum \psi_{2,i} \cdot Q_{k,i}\right)$$

Section 4.3

The combination factor for snow loads is $\psi_{2,1}$ = 0.0. With this parameter, the design bending load is calculated to:

$$M_{fi,d} = \left[\left(4.32 + 5.0\right) + 0.0 \cdot 11.25\right] \cdot \frac{35.0^2}{8} = 1427.1 \text{ kNm}$$

2.2 *Calculation of the steel temperature*

EN 1993-1-2

The temperature increase of the steel section is calculated to:

Section 4.2.5.1

$$\Delta\theta_{a,t} = k_{sh} \cdot \frac{A_m/V}{c_a \cdot \rho_a} \cdot \dot{h}_{net,d} \cdot \Delta t = 1.0 \cdot \frac{40}{600 \cdot 7850} \cdot 5 \cdot \dot{h}_{net} = 4.25 \cdot 10^{-5} \cdot \dot{h}_{net}$$

where:

k_{sh} correction factor for the shadow effect (k_{sh} = 1.0)
Δt time interval (Δt = 5 seconds)
A_m/V section factor for the unprotected beam

$$A_m/V = 1/t = 1/0.025 = 40 \ 1/\text{m}$$

The net heat flux is calculated according to EN 1991 Part 1-2.

EN 1991-1-2

$$\dot{h}_{net} = \alpha_c \cdot \left(\theta_g - \theta_m\right) + \Phi \cdot \varepsilon_m \cdot \varepsilon_r \cdot \sigma \cdot \left(\left(\theta_g + 273\right)^4 - \left(\theta_m + 273\right)^4\right)$$

$$= 25 \cdot \left(\theta_g - \theta_m\right) + 3.969 \cdot 10^{-8} \cdot \left(\left(\theta_g + 273\right)^4 - \left(\theta_m + 273\right)^4\right)$$

Section 3.1

The standard temperature-time curve is used for getting the gas temperatures.

$$\theta_g = 20 \cdot 345 \cdot \log_{10}\left(8 \cdot t + 1\right)$$

Section 3.2.1

The steel temperature-time curve of the hollow section is shown in Figure 3:

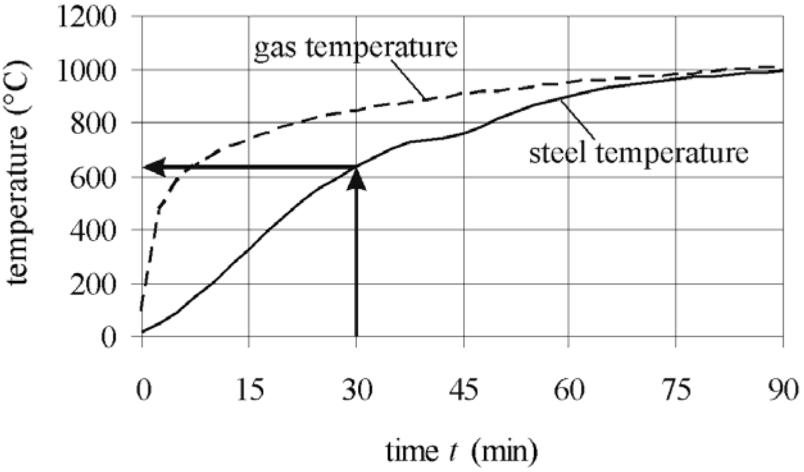


Figure 3. Steel temperature-time curve of the hollow section

$\Rightarrow \quad \theta_{a,max,30} = 646$ °C

2.3 *Verification in the temperature domain*

EN 1993-1-2

The design moment resistance during fire exposure at the time $t = 0$ is needed to get the utilization factor.

Section 4.2.3.3

$$M_{fi,Rd,0} = W_{pl} \cdot f_y \cdot k_{y,\theta,\max} / \gamma_{M,fi}$$

$$= 12{,}875{,}000 \cdot 355 \cdot \frac{1.0}{1.0} \cdot 10^{-6}$$

$$= 4570.6 \text{ kNm}$$

where:

$k_{y,\theta,max} = 1.0$ for $\theta = 20$ °C at the time $t = 0$

$\gamma_{M,fi} = 1.0$

and:

$$W_{pl} = 2 \cdot \left(\frac{2 \cdot A_w}{2} \cdot \frac{h_w}{4} + A_f \cdot \frac{h - t_w}{2} \right)$$

$$= 2 \cdot \left(16{,}250 \cdot \frac{650}{4} + 11{,}250 \cdot \frac{700 - 25}{2} \right)$$

$$= 12{,}875{,}000 \text{ mm}^3$$

The utilization factor is calculated to:

Section 4.2.4

$$\mu_0 = E_{fi,d} / R_{fi,d,0} = M_{fi,d} / M_{fi,Rd,0} = 1427.1/4570.6 = 0.31$$

The critical temperature $\theta_{a,cr}$ is given in Table 4.1 of the EN 1993 Part 1-2.

$\Rightarrow \theta_{a,cr} = 659$ °C

Verification:

$$\frac{646}{659} = 0.98 < 1 \qquad \checkmark$$

2.4 *Verification in the strength domain*

To calculate the moment resistance the reduction factor $k_{y,\theta}$ has to be determined for the temperature $\theta_{a,max,30}$ = 646 °C. This factor is given in Table 3.1 of the EN 1993 Part 1-2:

$$k_{y,\theta} = 0.360$$

Section 3.2.1

Additionally, the adaptation factors κ_1 and κ_2 have to be determined.

The adaptation factor κ_1 considers the non-uniform temperature distribution across the cross-section.

Table 1. Adaptation factor κ_1

Section 4.2.3.3

	κ_1 [-]
Beam exposed on all four sides	1.0
Unprotected beam exposed on three sides with a composite or concrete slab on side four	0.7
Protected beam exposed on three sides with a composite or concrete slab on side four	0.85

The beam in this is an unprotected beam exposed to fire on four sides. Therefore κ_1 is set to:

$$\kappa_1 = 1.0$$

The adaptation factor κ_2 considers the non-uniform temperature distribution along a beam.

Table 2. Adaptation factor κ_2

	κ_2 [-]
At the supports of a statically indeterminate beam	0.85
In all other cases	1.0

The verification is done in the middle of the beam and it is statically determinate. So the adaptation factor κ_2 is set to:

$$\kappa_2 = 1.0$$

Therefore the design moment resistance is calculated to:

$$M_{fi,t,Rd} = M_{pl,Rd,20°C} \cdot k_{y,\theta} \cdot \frac{\gamma_{M,1}}{\gamma_{M,fi}} \cdot \frac{1}{\kappa_1 \cdot \kappa_2}$$

$$= \left(12{,}87{,}000 \cdot 355/1.1\right) \cdot 0.36 \cdot \frac{1.1}{1.0} \cdot \frac{1}{1.0 \cdot 1.0} \cdot 10^{-6} = 1645.4 \text{ kNm}$$

Verification:

$$\frac{1427.1}{1645.4} = 0.87 < 1 \quad \checkmark$$

REFERENCES

EN 1991, *Eurocode 1:Actions on structures – Part 1-2: General actions – Actions on structures exposed to fire*, Brussels: CEN, November 2002

EN 1993, *Eurocode 3: Design of steel structures – Part 1-2: General rules – Structural fire design*, Brussels: CEN

Example to EN 1994 Part 1-2: Composite slab

P. Schaumann, T. Trautmann
University of Hannover – Institute for Steel Construction, Hannover, Germany

1 TASK

A composite slab has to be dimensioned in the fire situation. It is part of a shopping centre and the span is 4.8 m. The slab will be dimensioned as a series of simply supported beams. The required standard fire resistance class for the slab is R 90.

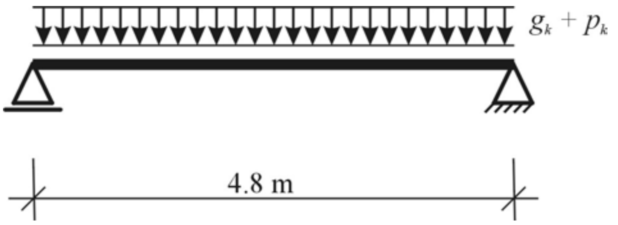


Figure 1. Static system

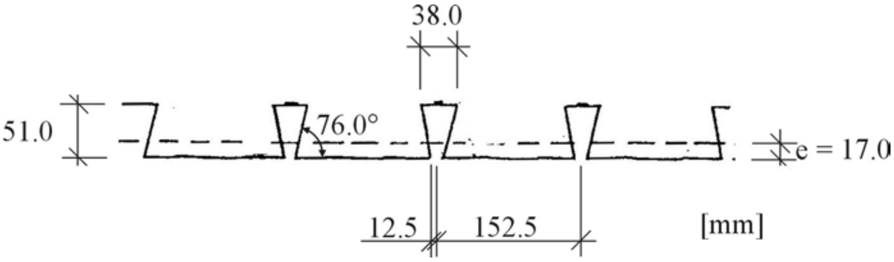


Figure 2. Steel sheet

Material properties
  Steel sheet:
    Yield stress: f_{yp} = 350 N/mm²
    Cross-sectional area: A_p = 1562 mm²/m
    Parameters for m+k method: k = 0.150 N/mm²
  Concrete:
    Strength category: C 25/30
    Compression strength: f_c = 25 N/mm²
    Height: h_t = 140 mm
    Cross-sectional area: A_c = 131,600 mm²/m

Scope of application for re-entrant profiles [mm]	Existing geometrical parameters [mm]
$77.0 \le l_1 \le 135.0$	$l_1 = 115.0$
$110 \le l_2 \le 150.0$	$l_2 = 140.0$
$38.5 \le l_3 \le 97.5$	$l_3 = 38.0$
$50.0 \le h_1 \le 130.0$	$h_1 = 89.0$
$30.0 \le h_2 \le 70.0$	$h_2 = 51.0$

Loads:
Permanent loads:
Steel sheet $g_{p,k}$ = 0.13 kN/m²
Concrete: $g_{c,k}$ = 3.29 kN/m²
Finishing load: $g_{f,k}$ = 1.2 kN/m²
Variable loads:
Live load: p_k = 5.0 kN/m²

Design sagging moment
at ambient temperatures: $M_{s,d}$ = 39.56 kNm

2 FIRE RESISTANCE OF A COMPOSITE SLAB

The composite slab has to be verified according to Section 4.3 and Annex D.

2.1 *Geometrical parameters and scope of application*

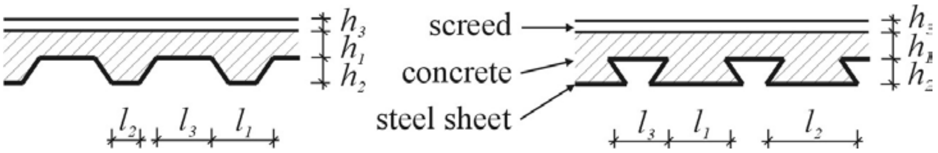


Figure 3. Geometry of cross-section

h_1 = 89 mm h_2 = 51 mm
l_1 = 115 mm l_2 = 140 mm l_3 = 38 mm

Table 1. Scope of application for slabs made of normal concrete and re-entrant steel sheets

Scope of application for re-entrant profiles [mm]	Existing geometrical parameters [mm]
$77.0 \le l_1 \le 135.0$	$l_1 = 115.0$
$110 \le l_2 \le 150.0$	$l_2 = 140.0$
$38.5 \le l_3 \le 97.5$	$l_3 = 38.0$
$50.0 \le h_1 \le 130.0$	$h_1 = 89.0$
$30.0 \le h_2 \le 70.0$	$h_2 = 51.0$

2.2 *Mechanical actions during fire exposure*

EN 1991-1-2

The load is determined by the combination rule for accidental situations.

$$E_{dA} = E\left(\sum G_k + A_d + \sum \psi_{2,i} \cdot Q_{k,i}\right)$$

Section 4.3

According to EN 1994 Part 1-2, the load E_d may be reduced by the reduction

EN 1994-1-2

factor η_{fi}. It is calculated to:

$$\eta_{fi} = \frac{G_k + \psi_{2,1} \cdot Q_{k,1}}{\gamma_G \cdot G_k + \gamma_{Q,1} \cdot Q_{k,1}} = \frac{(0.13+3.29+1.2)+0.6 \cdot 5.0}{1.35 \cdot (0.13+3.29+1.2)+1.5 \cdot 5.0} = 0.55$$

Section 2.4.2

With η_{fi}, the design bending moment $M_{fi,d}$ can be calculated:

$$M_{fi,d} = \eta_{fi} \cdot M_{sd} = 0.55 \cdot 39.56 = 21.94 \text{ kNm/m}$$

2.3 *Thermal insulation*

Section D.1

The thermal insulation criteria "I" has to ensure the limitation of the thermal condition of the member. The temperature on top of the slab should not exceed 140 °C in average and 180 °C at its maximum.

The verification is done in the time domain. The time in which the slab fulfils the criteria "I" is calculated to:

$$t_i = a_0 + a_1 \cdot h_1 + a_2 \cdot \Phi + a_3 \cdot \frac{A}{L_r} + a_4 \cdot \frac{1}{l_3} + a_5 \cdot \frac{A}{L_r} \cdot \frac{1}{l_3}$$

The rib geometry factor A/L_r is equivalent to the section factor A_p/V for beams. The factor considers that the mass and height have positive effects on the heating of the slab.

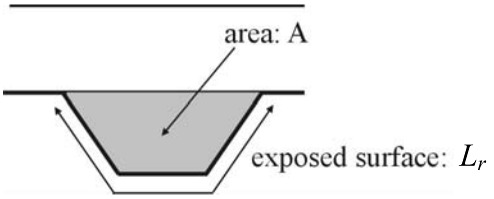


Figure 4. Definition of the rib geometry factor

$$\frac{A}{L_r} = \frac{h_2 \cdot \left(\frac{l_1 + l_2}{2}\right)}{l_2 + 2 \cdot \sqrt{h_2^{\,2} + \left(\frac{l_1 - l_2}{2}\right)^2}} = \frac{52 \cdot \left(\frac{115+140}{2}\right)}{140 + 2 \cdot \sqrt{51^2 + \left(\frac{115-140}{2}\right)^2}} = 27 \text{ mm}$$

The view factor Φ considers the shadow effect of the rib on the upper flange.

$$\Phi = \left[\sqrt{h_2^{\,2} + \left(l_3 + \frac{l_1 - l_2}{2}\right)^2} - \sqrt{h_2^{\,2} + \left(+\frac{l_1 - l_2}{2}\right)^2}\,\right] \Big/ l_3$$

$$= \left[\sqrt{51^2 + \left(38 + \frac{115-140}{2}\right)^2} - \sqrt{51^2 + \left(\frac{115-140}{2}\right)^2}\,\right] \Big/ 38$$

$$= 0.119$$

The coefficients a_i for normal weight concrete is given in Table 2:

Table 2. Coefficients for determination of the fire resistance with respect to thermal insulation (see EN 1994-1-2, Annex D, Table D.1)

	a_0 [min]	a_1 [min/mm]	a_2 [min]	a_3 [min/mm]	a_4 mm·min	a_5 [min]
Normal weight concrete	-28.8	1.55	-12.6	0.33	-735	48.0
Light weight concrete	-79.2	2.18	-2.44	0.56	-542	52.3

With these parameters, t_i is calculated to:

$$t_i = (-28.8) + 1.55 \cdot 89 + (-12.6) \cdot 0.119 + 0.33 \cdot 27 + (-735) \cdot 1/38 + 48 \cdot 27 \cdot 1/38 = 131.48 \text{ min} > 90 \text{ min} \quad \checkmark$$

2.4 *Verification of the load carrying-capacity*

Section 4.3.2

The plastic moment design resistance is calculated to:

$$M_{fi,t,Rd} = \sum A_i \cdot z_i \cdot k_{y,\theta,i} \cdot \left(\frac{f_{y,i}}{\gamma_{M,fi}} \right) + \alpha_{slab} \cdot \sum A_j \cdot z_j \cdot k_{c,\theta,j} \cdot \left(\frac{f_{c,j}}{\gamma_{M,fi,c}} \right)$$

To get the reduction factors $k_{y,\theta}$ for the upper flange, lower flange and the web, the temperatures have to be determined. These are calculated to:

Section D.2

$$\theta_a = b_0 + b_1 \cdot \frac{1}{l_3} + b_2 \cdot \frac{A}{L_r} + b_3 \cdot \Phi + b_4 \cdot \Phi^2$$

The coefficients b_i can be obtained from Table 3:

Table 3. Coefficients for the determination of the temperatures of the parts of the steel decking (see EN 1994-1-2, Annex D, Table D.2)

Concrete	Fire resistance [min]	Part of steel sheet	b_0 [°C]	b_1 [°C·mm]	b_2 [°C/mm]	b_3 [°C]	b_4 [°C]
Normal weight concrete	60	Lower flange	951	-1197	-2.32	86.4	-150.7
		Web	661	-833	-2.96	537.7	-351.9
		Upper flange	340	-3269	-2.62	1148.4	-679.8
	90	Lower flange	1018	-839	-1.55	65.1	-108.1
		Web	816	-959	-2.21	464.9	-340.2
		Upper flange	618	-2786	-1.79	767.9	-472.0
	120	Lower flange	1063	-679	-1.13	46.7	-82.8
		Web	925	-949	-1.82	344.2	-267.4
		Upper flange	770	-2460	-1.67	592.6	-379.0

For the different parts of the steel sheet, the temperatures are:
Lower flange:

$$\theta_{a,l} = 1018 - 839 \cdot \frac{1}{38} - 1.55 \cdot 27 + 65.1 \cdot 0.119 - 108.1 \cdot 0.119^2 = 960.29 \text{ °C}$$

Web:

$$\theta_{a,w} = 816 - 959 \cdot \frac{1}{38} - 2.21 \cdot 27 + 464.9 \cdot 0.119 - 340.2 \cdot 0.119^2$$
$$= 781.60 \text{ °C}$$

Upper flange:

$$\theta_{a,l} = 618 - 2786 \cdot \frac{1}{38} - 1.79 \cdot 27 + 767.9 \cdot 0.119 - 472.0 \cdot 0.119^2$$
$$= 580.87 \text{ °C}$$

To get the required load carrying-capacity during fire exposure, reinforcing bars have to be installed which normally are neglected for the ambient temperature design. For each rib, one reinforcing bar Ø 10 mm is chosen. The position of the bar can be seen in Figure 5.

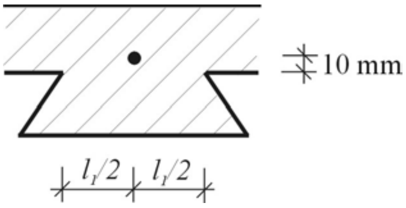


Figure 5. Arrangement of the reinforcing bar

The temperature of the reinforcing bar is calculated to:

$$\theta_s = c_0 + c_1 \cdot \frac{u_3}{h_2} + c_2 \cdot z + c_3 \cdot \frac{A}{L_r} + c_4 \cdot \alpha + c_5 \cdot \frac{1}{l_3}$$

where:

$$\frac{1}{z} = \frac{1}{\sqrt{u_1}} + \frac{1}{\sqrt{u_2}} + \frac{1}{\sqrt{u_3}}$$
$$= \frac{1}{\sqrt{l_1/2}} + \frac{1}{\sqrt{l_1/2}} + \frac{1}{\sqrt{h_2 + 10}} \quad \text{(simplified)}$$
$$= \frac{1}{\sqrt{57}} + \frac{1}{\sqrt{57}} + \frac{1}{\sqrt{61}}$$
$$= 0{,}393 \ 1/\text{mm}^{0.5}$$

$\Rightarrow \quad z = 2.54 \text{ mm}^{0.5}$

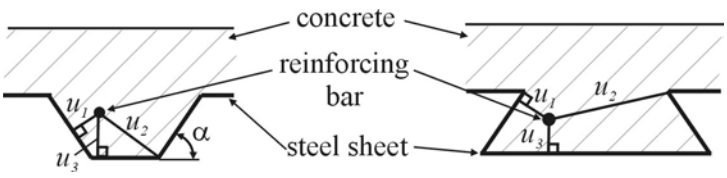


Figure 6. Definition of the distances u_1, u_2, u_3 and the angle α

The coefficients c_i for normal weight concrete is given in Table 4.

Table 4. Coefficients for the determination of the temperatures of the reinforcement bars in rib (see EN 1994-1-2, Annex D, Table D.3)

Concrete	Fire resistance [min]	c_0 [°C]	c_1 [°C]	c_2 [°C/mm$^{0.5}$]	c_3 [°C/mm]	c_4 [°C/°]	c_5 [°C]
Normal weight concrete	60	1191	-250	-240	-5.01	1.04	-925
	90	1342	-256	-235	-5.30	1.39	-1267
	120	1387	-238	-227	-4.79	1.68	-1326

With these parameters, the temperature of the reinforcing bar is:

$$\begin{aligned}\theta_s &= 1342+(-256)\cdot\frac{61}{51}+(-235)\cdot 2{,}54\\ &\quad +(-5{,}30)\cdot 27+1{,}39\cdot 104+(-1267)\cdot\frac{1}{38}\\ &= 407{,}0\ °\mathrm{C}\end{aligned}$$

For the steel sheet, the reduction factors $k_{\mathrm{y,i}}$ are given in Table 3.2 of the EN 1994-1-2. For the reinforcement the reduction factor is given in Table 3.4, because the reinforcement bars are cold worked.

The carrying-capacity for each part of the steel sheet and the reinforcing bars can now be calculated.

Table 5. Reduction factors and carrying-capacities

	Temperature θ_i [°C]	Reduction factor $k_{y,i}$ [-]	Partial area A_i [cm²]	$f_{y,i}$ [kN/cm²]	Z_i [kN]
Lower flange	960,29	0,047	1,204	35,0	1,98
Web	781,60	0,132	0,904	35,0	4,18
Upper flange	580,87	0,529	0,327	35,0	6,05
Reinforcement	407,0	0,921	0,79	50,0	36,38

The plastic neutral axis is calculated as equilibrium of the horizontal forces. The equilibrium is set up for one rib ($b = l_1 + l_2$).

$$z_{pl}=\frac{\sum Z_i}{a_{slab}\cdot(l_1+l_3)\cdot f_c}=\frac{1{,}98+4{,}18+6{,}05+36{,}38}{0{,}85\cdot(115+38)\cdot 25\cdot 10^{-3}}=15{,}0\ \mathrm{mm}$$

The plastic moment resistance for one rib is determined to:

Table 6. Calculation of the moment resistance of one rib

	Z_i [kN]	z_i [cm]	M_i [kNcm]
Lower flange	1,98	14,0	27,72
Web	4,18	14,0 – 5,1 / 2 = 11,45	47,86
Upper flange	6,05	14,0 – 5,1 = 8,9	53,85
Reinforcement	36,38	14,0 – 5,1 – 1,0 = 7,9	287,4
Concrete	-48,59	1,50 / 2 = 0,75	-36,44
			Σ 380,39

With the plastic moment of $M_{pl,rib}$ = 3.80 kNm and the width w_{rib} = 0.152 m of one rib, the plastic moment resistance of the composite slab is:

$$M_{fi,Rd}=3{,}80/0{,}152=25{,}00\ \mathrm{kNm/m}$$

Verification:

$$\frac{21,94}{25,00} = 0,88 < 1 \qquad \checkmark$$

REFERENCES

EN 1991, *Eurocode 1:Actions on structures – Part 1-2: General actions – Actions on structures exposed to fire*, Brussels: CEN, November 2002
EN 1994, *Eurocode 4: Design of composite steel and concrete structures – Part 1-2: General Rules – Structural Fire Design*, Brussels: CEN, October 2003

Example to EN 1994 Part 1-2: Composite beam

P. Schaumann, T. Trautmann
University of Hannover – Institute for Steel Construction, Hannover, Germany

1 TASK

A fire safety verification has to be done for a composite beam of an office building. It is a simply supported beam and is loaded uniformly. The concrete slab of the composite beam protects the beam from the top in the fire situation, so the steel beam is exposed to fire on three sides. For fire protection of the steel beam a contour encasement of plaster is chosen. The required standard fire resistance class for the beam is R 60.

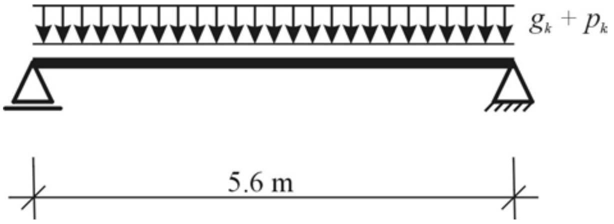


Figure 1. Static system

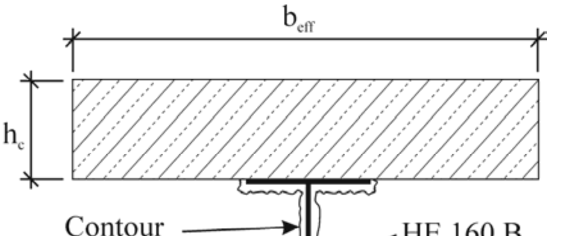


Figure 2. Cross-section

Material properties:
- Beam:
  - Profile: rolled section HE 160 B
  - Steel grade: S 355
  - Height: $h = 160$ mm
  - Height of web: $h_w = 134$ mm
  - Width: $b = b_1 = b_2 = 160$ mm

Thickness of web: e_w = 8 mm
Thickness of flange: e_f = e_1 = e_2 = 13 mm
Cross-sectional area: A_a = 5430 mm²
Yield stress: $f_{y,a}$ = 355 N/mm²

Slab:
Strength category: C 25/30
Height: h_c = 160 mm
Effective width: b_{eff} = 1400 mm
Compression strength: f_c = 25 N/mm²
Elastic modulus: E_{cm} = 29,000 N/mm²

Shear connectors:
Quantity: n = 34 (equidistant)
Diameter: d = 22 mm
Tensile strength: f_u = 500 N/mm²

Encasement:
Material: plaster
Thickness: d_p = 15 mm (contour encasement)
Thermal conductivity: λ_p = 0.12 W/(m·K)
Specific heat: c_p = 1100 J/(kg·K)
Density: ρ_p = 550 kg/m³

Loads:
Permanent loads:
Self weight: g_k = 20.5 kN/m
Finishing load: g_k = 7.5 kN/m
Variable loads:
Live load: p_k = 15.0 kN/m

2 FIRE RESISTANCE OF A COMPOSITE BEAM

2.1 *Mechanical actions during fire exposure*

EN 1991-1-2

Actions on structures from fire exposure are classified as accidental situation:

Section 4.3

$$E_{dA} = E\left(\sum G_k + A_d + \sum \psi_{2,i} \cdot Q_{k,i}\right)$$

The partial safety factor γ_{GA} for the accidental situation is $\gamma_{GA} = 1.0$. The combination factor for the leading variable action for office buildings is set to $\psi_{2,1} = 0.3$.

With these parameters, the design bending moment during fire exposure can be calculated:

$$M_{fi,d} = \left(20.5 + 7.5\right) + 0.3 \cdot \left(15.0\right) \cdot \frac{5.6^2}{8} = 127.4 \text{ kNm}$$

2.2 *Calculation of the temperatures in the cross-section*

EN 1994-1-2

For the calculation of the temperatures, the cross-section is split into different sections. These are the concrete slab, the upper flange, the web and the lower flange. It is done according to Section 4.3.4.2 of EN 1994-1-2.

The temperatures of the upper flange, the web and the lower flange are determined by using the Euro-Nomogram (“Euro-Nomogram”, ECCS No.89, 1996). Therefore, the section factors of these parts are required.

Lower flange:

$$\left(\frac{A_p}{V}\right)_l = \frac{2\cdot(b_1+e_1)}{b_1\cdot e_1} = \frac{2\cdot(0.16+0.013)}{0.16\cdot 0.013} = 166.3\ \text{m}^{-1}$$

Section 4.3.4.2

Web:

$$\left(\frac{A_p}{V}\right)_w = \frac{2\cdot(h_w)}{h_w\cdot e_w} = \frac{2\cdot(0.134)}{0.134\cdot 0.008} = 250.0\ \text{m}^{-1}$$

Upper flange (more than 85% of the upper flange is in contact with the concrete slab):

$$\left(\frac{A_p}{V}\right)_u = \frac{(b_2+2\cdot e_2)}{b_2\cdot e_2} = \frac{(0.16+2\cdot 0.013)}{0.16\cdot 0.013} = 89.4\ \text{m}^{-1}$$

The temperatures are determined to:

ECCS No.89

Table 1. Temperatures of upper flange, web and lower flange

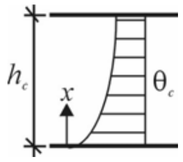
	$\left(\frac{A_p}{V}\right)_i \cdot \frac{\lambda_p}{d_p}$ $\left[\frac{\text{W}}{\text{m}^3\text{K}}\right]$	$\theta_{a,max,60}$ [°C]
Upper flange	715	390
Web	2000	650
Lower flange	1330	550

The temperature of the concrete slab is not constant over its thickness. Therefore the compression strength varies over the thickness. For temperatures lower than 250 °C, the compression strength is not reduced. Above 250 °C it is reduced by the reduction factor $k_{c,\theta}$. Assessment of the temperatures may be done in layers of 10 mm thickness on basis of Table 2.

EN 1994-1-2
Section D.3

Table 2. Temperature distribution in a solid slab of 100 mm thickness composed of normal weight concrete and not insulated (see EN 1994-1-2, Annex D.3, Table D.5)

Depth	Temperature θ_c [°C] after a fire duration in min. of					
x [mm]	30’	60’	90’	120’	180’	240’
5	535	705				
10	470	642	738			
15	415	581	681	754		
20	350	525	627	697		
25	300	469	571	642	738	
30	250	421	519	591	689	740
35	210	374	473	542	635	700
40	180	327	428	493	590	670
45	160	289	387	454	549	645
50	140	250	345	415	508	550
55	125	200	294	369	469	520
60	110	175	271	342	430	495
80	80	140	220	270	330	395
100	60	100	160	210	260	305



2.3 *Verification using simple calculation model*

The composite beam is verified by the simple calculation model. It is done in the strength domain. The calculation of the moment resistance is accomplished according to Annex E.

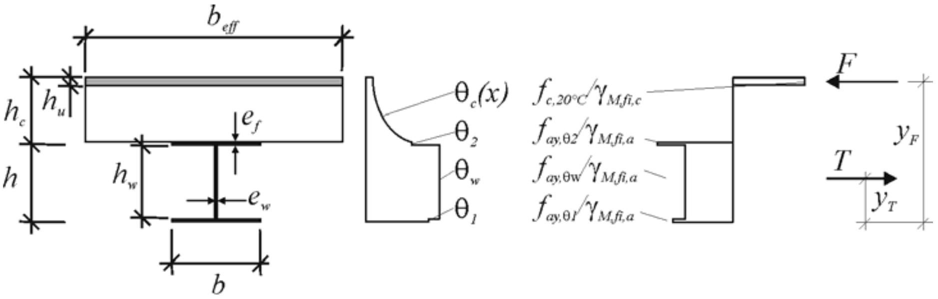


Figure 3. Calculation of the moment resistance

The temperatures of the parts of the steel beam were determined in Section 3.2. The reduction factors $k_{y,\theta,i}$ for the calculation of the yield stresses at elevated temperatures, are given in Table 3.2 of EN 1994-1-2, Section 3.2.1.

Table 3. Calculation of the reduced yield stresses

	$\theta_{a,max,60}$ [°C]	$k_{y,\theta}$ [-]	$f_{ay,\theta}$ [kN/cm²]
Upper flange	390	1.00	35.5
Web	650	$(0.47+0.23)/2=0.35$	12.4
Lower flange	550	$(0.78+0.47)/2=0.625$	22.2

Section E.1

The next step is the calculation of the tensile force T of the steel beam according to Figure 3.

$$T=\frac{f_{ay,\theta 1}\cdot\left(b\cdot e_f\right)+f_{ay,\theta w}\cdot\left(h_w\cdot e_w\right)+f_{ay,\theta 2}\cdot\left(b\cdot e_f\right)}{\gamma_{M,fi,a}}$$

$$=\frac{22.2\cdot(16\cdot 1.3)+12.4\cdot(13.4\cdot 0.8)+35.5\cdot(16\cdot 1.3)}{1.0}$$

$$=1333.1\text{ kN}$$

The location of the tensile force is determined to:

$$y_T=\frac{f_{ay,\theta 1}\cdot\left(b\cdot\frac{e_f^2}{2}\right)+f_{ay,\theta w}\cdot\left(h_w\cdot e_w\right)\cdot\left(e_f+\frac{h_w}{2}\right)+f_{ay,\theta 2}\cdot\left(b\cdot e_f\right)\left(h-\frac{e_f}{2}\right)}{T\cdot\gamma_{M,fi,a}}$$

$$=\frac{22.2\cdot\left(16\cdot\frac{1.3^2}{2}\right)+12.4\cdot(13.4\cdot 0.8)\cdot\left(1.3+\frac{13.4}{2}\right)+35.5\cdot(16\cdot 1.3)\cdot\left(16-\frac{1.3}{2}\right)}{1333.1\cdot 1.0}$$

$$=9.53\text{ cm}$$

In a simply supported beam, the value of the tensile force T is limited by:

$$T \leq N \cdot P_{fi,Rd}$$

where:

N Number of shear connectors in one of the critical lengths of the beam

$P_{fi,Rd}$ Design resistance in the fire situation of a shear connector

To get $P_{fi,Rd}$, the reduction factors $k_{u,\theta}$ and $k_{c,\theta}$ (Table 5) as well as the design resistances of a shear connector at ambient temperatures $P_{Rd,1}$ and $P_{Rd,2}$ are needed.

The temperatures for getting the reduction factors are determined as 80 % (stud connector) and 40 % (concrete) of the steel flange (see EN 1994 Part 1-2, Section 4.3.4.2.5 (2)). The reduction factor for the tensile strength of the stud connector is given it Table 3.2 of EN 1994-1-2, Section 3.2.1. The reduction factor for the compression strength of the concrete is given in Table 3.3 of EN 1994-1-2, Section 3.2.1.

$$\theta_v = 0.8 \cdot 390 = 312 \text{ °C}$$

$$\Rightarrow \quad k_{u,\theta} = 1.0$$

$$\theta_c = 0.4 \cdot 390 = 156 \text{ °C}$$

$$\Rightarrow \quad k_{c,\theta} = 0.98$$

The design resistances of the shear connector are calculated according to EN 1994-1-1, with the partial safety factor $\gamma_{M,fi,v}$ replacing γ_v

EN 1994-1-1

$$P_{Rd,1} = 0.8 \cdot \frac{f_u}{\gamma_{M,fi,v}} \cdot \frac{\pi \cdot d^2}{4} = 0.8 \cdot \frac{50.0}{1.0} \cdot \frac{\pi \cdot 2.2^2}{4} = 152 \text{ kN}$$

Section 6.3.2.1

$$P_{Rd,2} = 0.29 \cdot \alpha \cdot d^2 \cdot \frac{\sqrt{f_c \cdot E_{cm}}}{\gamma_{M,fi,v}} = 0.29 \cdot 1.0 \cdot 2.2^2 \cdot \frac{\sqrt{2.5 \cdot 2900}}{1.0} = 120 \text{ kN}$$

The design resistance in the fire situation of a shear connector is:

EN 1994-1-2

$$P_{fi,Rd} = \min \begin{cases} P_{fi,Rd,1} = 0.8 \cdot k_{u,\theta} \cdot P_{Rd,1} = 0.8 \cdot 1.0 \cdot 152 = 121.6 \text{ kN} \\ P_{fi,Rd,2} = k_{c,\theta} \cdot P_{Rd,2} = 0.98 \cdot 120 = 117.6 \text{ kN} \quad \leftarrow \text{ relevant} \end{cases}$$

Section 4.3.4.2

So, the limitation can be verified:

$$1333.1 \text{ kN} < 34/2 \cdot 117.6 = 1999.2 \text{ kN}$$

Section E.1

For equilibrium of forces, the compression force has to be equal to the tension force. Therefore the thickness of the compressive zone h_u is determined to:

$$h_u = \frac{T}{b_{eff} \cdot f_c / \gamma_{M,fi,c}} = \frac{1333.1}{140.0 \cdot 2.5/1.0} = 3.8 \text{ cm}$$

Now, two situations may occur. The first one is that the temperature in every layer of the concrete in the compression zone is lower than 250 °C. In the second situation the temperature of some layers of the concrete is above 250 °C. To check which situation occurs, following calculation has to be done:

$$\left(h_c - h_u\right) = 16 - 3.8 = 12.2 \text{ cm}$$

If the result of this equation is greater than the depth x according to Table 2, corresponding to a concrete temperature below 250 °C, the concrete in the compression zone may not be reduced.

$$h_{cr} = x = 5.0 \text{ cm} < 12.2 \text{ cm}$$

The point of application of the compression force y_F is determined to:

$$y_F = h + h_c - \left(h_u/2\right) = 16 + 16 - \left(3.8/2\right) = 30.1 \text{ cm}$$

The moment resistance is calculated to:

$$M_{fi,Rd} = T \cdot \left(y_F - y_T\right) = 1333.1 \cdot \left(30.1 - 9.53\right) \cdot 10^{-2} = 274.2 \text{ kNm}$$

Verification:

$$127.4/274.2 = 0.46 < 1 \qquad \checkmark$$

REFERENCES

ECCS No.89, *Euro-Nomogram*, Brussels: ECCS – Technical Committee 3 – Fire Safety of Steel Structures, 1995

EN 1991, *Eurocode 1:Actions on structures – Part 1-2: General actions – Actions on structures exposed to fire*, Brussels: CEN, November 2002

EN 1994, *Eurocode 4: Design of composite steel and concrete structures – Part 1-1: General Rules and rules for buildings*, Brussels: CEN

EN 1994, Eurocode *4: Design of composite steel and concrete structures – Part 1-2: General Rules – Structural Fire Design*, Brussels: CEN

Example to EN 1994 Part 1-2: Composite beam comprising steel beam with partial concrete encasement

P. Schaumann, T. Trautmann
University of Hannover – Institute for Steel Construction, Hannover, Germany

1 TASK

A fire safety verification has to be done for a composite beam of a storehouse. It is a simply supported beam with a uniform load and has a span of 12.0 m. The steel beam is partially encased and the slab is a composite slab. The required standard fire resistance class for the beam is R 90.

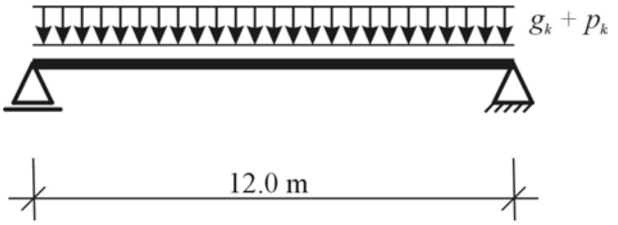


Figure 1. Static system

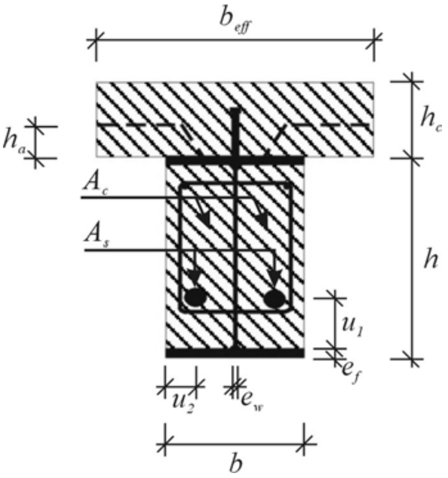


Figure 2. Cross-section

Material properties:

- Beam:
  - Profile: rolled section IPE 500
  - Steel grade: S 355
  - Height: $h = 500$ mm
  - Width: $b = 200$ mm
  - Thickness of web: $e_w = 10.2$ mm
  - Thickness of flange: $e_f = 16$ mm
  - Cross-sectional area: $A_a = 11{,}600$ mm²
  - Yield stress: $f_{y,a} = 355$ N/mm²
- Slab:
  - Strength category: C 25/30
  - Height: $h_c = 160$ mm
  - Effective width: $b_{eff} = 3000$ mm
  - Compression strength: $f_c = 25$ N/mm²
- Profiled steel sheet:
  - Type: re-entrant
  - Height: $h_a = 51$ mm
- Reinforcement in partial concrete encasement:
  - Steel grade: S 500
  - Diameter: 2 Ø 30
  - Cross-sectional area: $A_s = 1410$ mm²
  - Axis distances: $u_1 = 110$ mm
    $u_{s1} = 60$ mm
  - Yield stress: $f_{y,s} = 500$ N/mm²
- Concrete between flanges:
  - Strength category: C 25/30
  - Width: $b_c = 200$ mm
  - Compression strength: $f_c = 25$ N/mm²

Loads:

- Permanent loads:
  - Self-weight: $g_{s,k} = 15.0$ kN/m
  - Finishing load: $g_{f,k} = 6.0$ kN/m
- Variable loads:
  - Live load: $p_k = 30.0$ kN/m

2 FIRE RESISTANCE OF A COMPOSITE BEAM COMPRISING STEEL BEAM WITH PARTIAL CONCRETE ENCASEMENT

2.1 *Mechanical actions during fire exposure*

EN 1991-1-2

Actions on structures in fire situation are classified as an accidental situation:

$$E_{dA} = E\left(\sum G_k + A_d + \sum \psi_{2,i} \cdot Q_{k,i}\right)$$

Section 4.3

The combination factor for the leading variable action and for a storehouse is $\psi_{2,1} = 0.8$.

With these parameters, the design bending moment during fire exposure can be calculated:

$$M_{fi,d} = \left(\left(15.0 + 6.0\right) + 0.8 \cdot \left(30.0\right)\right) \cdot \frac{12.0^2}{8} = 810.0 \text{ kNm}$$

2.2 *Verification using simple calculation model*

EN 1994-1-2

The composite beam is verified by the simple calculation model. It is accomplished according to EN 1994 Part 1-2, Section 4.3.4.3 and Annex F.

To use this model, the slab should have a minimum thickness h_c. Additionally the steel beam should have a minimum height h, a minimum width b_c (where b_c is the minimum width of steel beam or concrete encasement) and a minimum area $h \cdot b_c$ (see Table 1).

Section 4.3.4.3

Table 1. Minimum dimensions for the use of the simple calculation model for composite beams comprising steel beams with partial concrete encasement (see EN 1994 Part 1-2, Section 4.3.4.3, Tables 4.9 and 4.10)

Standard fire resistance	Minimum slab thickness h_c [mm]	Minimum profile height h and minimum width b_c [mm]	Minimum area $h \cdot b_c$ [mm²]
R 30	60	120	17,500
R 60	80	150	24,000
R 90	100	170	35,000
R 120	120	200	50,000
R 180	150	250	80,000

$$h_c = 160 \text{ mm} > \min h_c = 100 \text{ mm} \qquad ✓$$

$$h = 500 \text{ mm} > \min h = 170 \text{ mm} \qquad ✓$$

$$b = b_c = 200 \text{ mm} > \min b_c = 170 \text{ mm} \qquad ✓$$

$$h \cdot b_c = 100{,}000 \text{ mm} > \min\left(h \cdot b_c\right) = 35{,}000 \text{ mm} \qquad ✓$$

In the calculation model of Annex F, the cross section is divided into different parts. At some parts the yield stress at other parts the cross-sectional area is reduced.

Section F.1

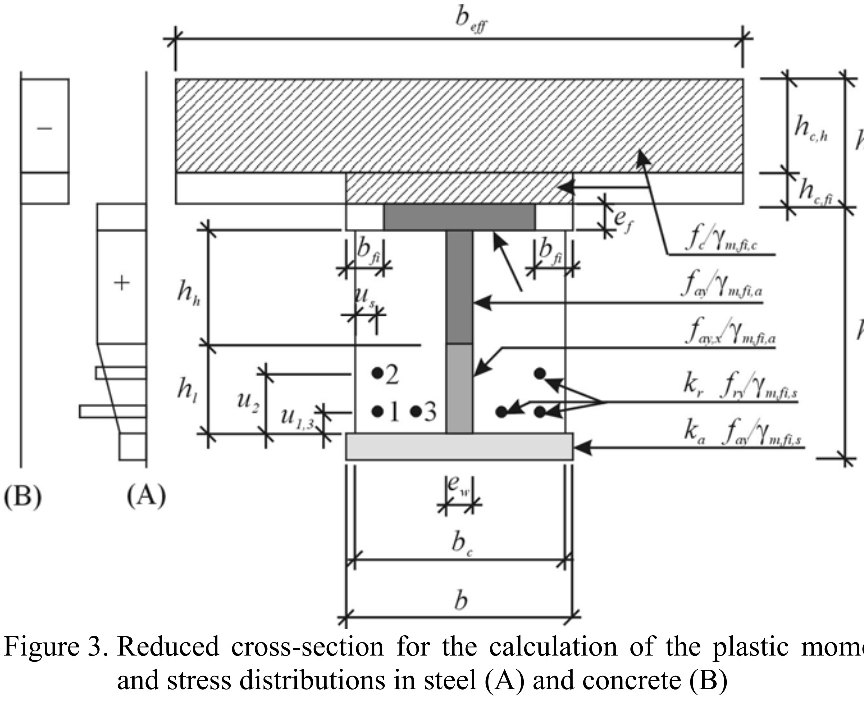


Figure 3. Reduced cross-section for the calculation of the plastic moment resistance and stress distributions in steel (A) and concrete (B)

The heating of the concrete slab is considered by reducing the cross-sectional area. For the different fire resistance classes, the thickness reduction $h_{c,fi}$ is given in Table 2. For composite slabs made with re-entrant steel sheets, a minimum thickness reduction $h_{c,fi,min}$ has to be considered. This minimum thickness reduction is equal to the height of the steel sheet.

$$h_{c,fi} = 30 \text{ mm}$$

$$h_{c,fi,min} = 51 \text{ mm}$$

For this, the height of the concrete during fire exposure $h_{c,h}$ is:

$$h_{c,h} = 160 - 51 = 109 \text{ mm}$$

Table 2. Thickness reduction $h_{c,fi}$ of the concrete slab (see EN 1994 Part 1-2, Annex F, Table F.1)

Standard fire resistance	Slab reduction $h_{c,fi}$ [mm]
R 30	10
R 60	20
R 90	30
R 120	40
R 180	55

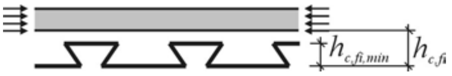


Figure 4. Minimum thickness $h_{c,fi,min}$ reduction for re-entrant profiles

The heating of the upper flange of the steel beam is considered by reducing its cross-sectional area. The calculation of the width reduction b_{fi} is shown in Table 3.

$$b_{fi} = (16.0/2) + 30 + (200 - 200)/2 = 38.0 \text{ mm}$$

The effective width is calculated to:

$$b_{fi,u} = 200 - 2 \cdot 38 = 124.0 \text{ mm}$$

Table 3. Width reduction b_{fi} of the upper flange (see EN 1994 Part 1-2, Annex F, Table F.2)

Standard fire resistance	Width reduction b_{fi} [mm]
R 30	$(e_f/2) + (b - b_c)/2$
R 60	$(e_f/2) + 10 + (b - b_c)/2$
R 90	$(e_f/2) + 30 + (b - b_c)/2$
R 120	$(e_f/2) + 40 + (b - b_c)/2$
R 180	$(e_f/2) + 60 + (b - b_c)/2$

The web of the steel beam is divided into two parts. The upper part of the web possesses the full yield stress, where the yield stress of the lower part has a linear gradient, from the yield stress of the upper part to the reduced yield stress of the lower flange. The height of the lower part of the web h_l is calculated to:

$$h_l = \frac{a_1}{b_c} + \frac{a_2 \cdot e_w}{b_c \cdot h} > h_{l,\min}$$

The parameters a_1 and a_2, as well as the minimum height $h_{l,min}$, are given in Table 4 for $h/b_c > 2$.

$$h_l = \frac{14{,}000}{200} + \frac{75{,}000 \cdot 10.2}{200 \cdot 500} = 77.7 \text{ mm} > 40 \text{ mm}$$

Table 4. Parameters a_1, a_2 and minimum height $h_{l,min}$ for $h/b_c > 2$ (see EN 1994 Part 1-2, Annex F, Table F.3)

Standard fire resistance	a_1 [mm²]	a_2 [mm²]	$h_{l,min}$ [mm]
R 30	3600	0	20
R 60	9500	0	30
R 90	14,000	75,000	40
R 120	23,000	110,000	45
R 180	35,000	250,000	55

The lower flange is not reduced by its cross-sectional area. Here, the yield stress is reduced by the factor k_a. This factor is limited by a minimum and maximum value. These limits, as well as the calculation of k_a, are given in Table 5.

$$a_0 = 0.018 \cdot e_f + 0.7 = 0.018 \cdot 16.0 + 0.7 = 0.988$$

$$k_a = \left(0.12 - \frac{17}{200} + \frac{500}{38 \cdot 200}\right) \cdot 0.988 = 0.100 \begin{cases} > 0.06 \\ < 0.12 \end{cases}$$

Table 5. Reduction factor k_a of the yield stress of the lower flange (see EN 1994 Part 1-2, Annex F, Table F.4)

Standard fire resistance	Reduction factor k_a	$k_{a,min}$	$k_{a,max}$
R 30	$\left(1.12 - \frac{84}{b_c} + \frac{h}{22 \cdot b_c}\right) \cdot a_0$	0.5	0.8
R 60	$\left(0.21 - \frac{26}{b_c} + \frac{h}{24 \cdot b_c}\right) \cdot a_0$	0.12	0.4
R 90	$\left(0.12 - \frac{17}{b_c} + \frac{h}{38 \cdot b_c}\right) \cdot a_0$	0.06	0.12
R 120	$\left(0.1 - \frac{15}{b_c} + \frac{h}{40 \cdot b_c}\right) \cdot a_0$	0.05	0.1
R 180	$\left(0.03 - \frac{3}{b_c} + \frac{h}{50 \cdot b_c}\right) \cdot a_0$	0.03	0.06

The heating of the reinforcing bars in the partial concrete encasement is considered by reducing the yield stress. The reduction factor depends on the fire resistance class and the position of the reinforcing bars. Like the reduction factor k_a, the reduction factor k_r has an upper and lower limit.

$$A_m = 2 \cdot h + b_c = 2 \cdot 500 + 200 = 1200 \text{ mm}$$

$$V = h \cdot b_c = 500 \cdot 200 = 100{,}000 \text{ mm}^2$$

$$\begin{aligned} u &= \frac{1}{(1/u_i) + (1/u_{si}) + 1/(b_c - e_w - u_{si})} \\ &= \frac{1}{(1/110) + (1/60) + 1/(200 - 10.2 - 60)} \\ &= 29.88 \text{ mm} \end{aligned}$$

$$k_r = \frac{(u \cdot a_3 + a_4) \cdot a_5}{\sqrt{A_m / V}} = \frac{(29.88 \cdot 0.026 - 0.154) \cdot 0.09}{\sqrt{1200/100{,}000}} = 0.51 \begin{cases} > 0.1 \\ < 1.0 \end{cases}$$

Table 6. Parameters for calculation of k_r (see EN 1994 Part 1-2, Annex F, Table F.5)

Standard fire resistance	a_3	a_4	a_5	$k_{r,min}$	$k_{r,max}$
R 30	0.062	0.16	0.126	0.1	1.0
R 60	0.034	-0.04	0.101	0.1	1.0
R 90	0.026	-0.154	0.090	0.1	1.0
R 120	0.026	-0.284	0.082	0.1	1.0
R 180	0.024	-0.562	0.076	0.1	1.0

To acquire the plastic moment resistance, the axial forces of the different parts should be determined.

Concrete:

$$C_c = b_{eff} \cdot h_{c,h} \cdot \alpha_c \cdot f_c = 300.0 \cdot 10.9 \cdot 0.85 \cdot 2.5 = 6948.8 \text{ kN}$$

Upper flange:

$$T_{f,u} = b_{fi,u} \cdot e_f \cdot f_y = 12.4 \cdot 1.60 \cdot 35.5 = 704.3 \text{ kN}$$

Upper web:

$$T_{w,u} = e_w \cdot h_h \cdot f_y = 1.02 \cdot 39.03 \cdot 35.5 = 1413.3 \text{ kN}$$

where:

$$h_h = h - 2 \cdot e_f - h_l = 50.0 - 2 \cdot 1.6 - 7.77 = 39.03 \text{ cm}$$

Lower web:

$$T_{w,l} = e_w \cdot h_l \cdot \left(\frac{1 + k_a}{2}\right) \cdot f_y = 1.02 \cdot 7.77 \cdot \left(\frac{1 + 0.1}{2}\right) \cdot 35.5 = 154.7 \text{ kN}$$

$$z_{w,l} = h_l \cdot \frac{2 \cdot k_a + 1}{3 \cdot (k_a + 1)} = 7.77 \cdot \frac{2 \cdot 0.1 + 1}{3 \cdot (0.1 + 1)} = 2.8 \text{ cm}$$

Lower flange:

$$T_{f,l} = b \cdot e_f \cdot k_a \cdot f_{y,a} = 20.0 \cdot 1.6 \cdot 0.1 \cdot 35.5 = 113.6 \text{ kN}$$

Reinforcement bars:

$$T_r = A_s \cdot k_r \cdot f_{y,s} = 14.1 \cdot 0.51 \cdot 50.0 = 359.6 \text{ kN}$$

Due to the fact that the compression force C_c is larger than the sum of the tension forces T_i, the plastic neutral axis is situated in the concrete slab. So the plastic neutral axis is calculated to:

$$z_{pl} = \frac{\sum T_i}{\alpha_c \cdot f_c \cdot b_{eff}} = \frac{704.3 + 1413.3 + 154.7 + 113.6 + 359.6}{0.85 \cdot 2.5 \cdot 300} = 4.31 \text{ cm}$$

To get the moment resistance, the lever arms of the forces are needed:
Concrete slab (referring to upper edge of slab):

$$z_c = z_{pl}/2 = 4.31/2 = 2.16 \text{ cm}$$

Upper flange (referring to centre of gravity of concrete slab):

$$z_{f,u} = h_c + e_f/2 - z_c = 16.0 + 1.6/2 - 2.16 = 14.64 \text{ cm}$$

Upper web:

$$z_{w,u} = h_c + e_f + h_h/2 - z_c = 16.0 + 1.6 + 39.03/2 - 2.16 = 34.96 \text{ cm}$$

Lower web:

$$z_{w,l} = h_c + e_f + h_h + z_{w,l} - z_c = 16 + 1.6 + 39.03 + 2.8 - 2.16 = 57.27 \text{ cm}$$

Lower flange:

$$z_{f,l} = h_c + h - e_f/2 - z_c = 16.0 + 50.0 - 1.6/2 - 2.16 = 63.04 \text{ cm}$$

Reinforcement:

$$z_r = h_c + h - e_f - u_1 - z_c = 16.0 + 50.0 - 1.6 - 11.0 - 2.16 = 51.24 \text{ cm}$$

The plastic moment resistance is determined to:

$$\begin{aligned} M_{fi,Rd} &= T_{f,u} \cdot z_{f,u} + T_{w,u} \cdot z_{w,u} + T_{w,l} \cdot z_{w,l} + T_{f,l} \cdot z_{f,l} + T_r \cdot z_r \\ &= 704.3 \cdot 14.64 + 1413.3 \cdot 34.96 + 154.7 \cdot 57.27 + 113.6 \cdot 63.04 \\ &\quad + 359.6 \cdot 51.24 \\ &= 10{,}311 + 49{,}409 + 8860 + 7161 + 18{,}426 \\ &= 94{,}167 \text{ kNcm} = 942.7 \text{ kNm} \end{aligned}$$

Verification:

$$\frac{810.0}{942.7} = 0.86 < 1 \qquad \checkmark$$

REFERENCES

EN 1991, *Eurocode 1:Actions on structures – Part 1-2: General actions – Actions on structures exposed to fire*, Brussels: CEN, November 2002
EN 1994, *Eurocode 4: Design of composite steel and concrete structures – Part 1-2: General Rules – Structural Fire Design*, Brussels: CEN

Example to EN 1994 Part 1-2: Composite column with partially encased steel sections

P. Schaumann, T. Trautmann
University of Hannover – Institute for Steel Construction, Hannover, Germany

1 TASK

The following example deals with a composite column made of partially encased steel sections. It is part of an office building and has a length of $L = 4.0$ m. In this example, the simple calculation model and the “tabulated data” method are used. The column is part of a braced frame and is connected bending resistant to the upper and lower column. Therefore the buckling length can be reduced as seen in Figure 1. The required standard fire resistance class for the column is R 60.

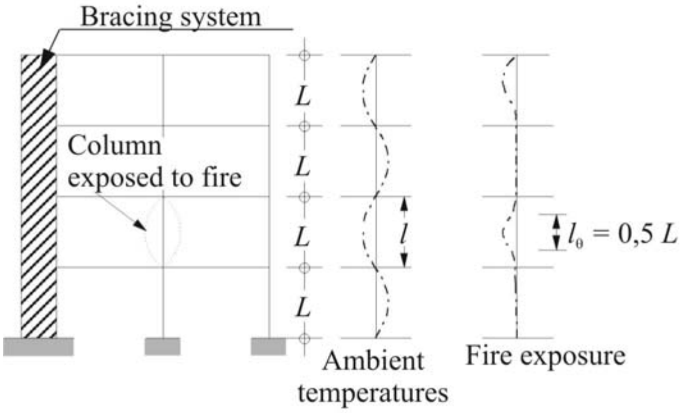


Figure 1. Buckling lengths of columns in braced frames

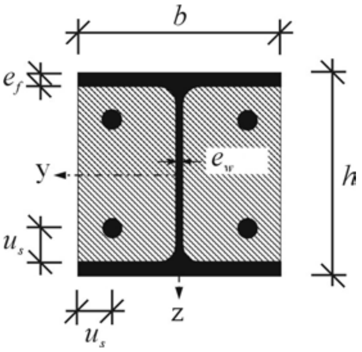


Figure 2. Cross-section of the column

Material properties:
Steel column:
Profile: rolled section HE 300 B
Steel grade: S 235
Height: $h = 300$ mm
Width: $b = 300$ mm
Thickness of web: $e_w = 11$ mm
Thickness of flange: $e_f = 19$ mm
Cross-sectional area: $A_a = 14900$ mm²
Yield stress: $f_y = 235$ N/mm²
Elastic modulus: $E_a = 210{,}000$ N/mm²
Moment of inertia: $I_z = 8560$ cm^4 (weak axis)
Reinforcement:
Steel grade: S 500
Diameter: 4 Ø 25
Cross-sectional area: $A_s = 1960$ mm²
Yield stress: $f_s = 500$ N/mm²
Elastic modulus: $E_s = 210{,}000$ N/mm²
Moment of inertia: $I_s = 4 \cdot 4.9 \cdot (30.0 / 2 - 5.0)^2 = 1960$ cm^4
Axis distance: $u_s = 50$ mm
Concrete:
Strength category: C 25/30
Cross-sectional area: $A_c = 300 \cdot 300 - 14900 - 1960 = 73{,}140$ mm²
Compression strength: $f_c = 25$ N/mm²
Elastic modulus: $E_{cm} = 30{,}500$ N/mm²
Moment of inertia: $I_c = 30 \cdot 30^3 / 12 - 8560 - 1960 = 56{,}980$ cm^4

Loads:
Permanent loads: $G_k = 960$ kN
Variable loads: $P_k = 612.5$ kN

2 FIRE RESISTANCE OF A COMPOSITE COLUMN WITH PARTIALLY ENCASED STEEL SECTIONS

2.1 *Mechanical actions during fire exposure*

EN 1991-1-2

For fire design the accidental situation for combining loads is used:

$$E_{dA} = E\left(\sum G_k + A_d + \sum \psi_{2,i} \cdot Q_{k,i}\right)$$

Section 4.3

With $\psi_{2,1} = 0.3$ the axial design load during fire exposure is:

$$N_{fi,d} = 1.0 \cdot 960 + 0.3 \cdot 612.5 = 1143.8 \text{ kN}$$

2.2 *Verification using simple calculation model*

prEN 1994-1-2

2.2.1 *Scope of application*

The simple calculation model is a verification in the strength domain. It has to be verified, that the load at elevated temperatures is smaller than the design resistance.

$$N_{fi,d} / N_{fi,Rd} \leq 1$$

Section 4.3.5.1

The design resistance for axial loads and buckling around the z-axis (weak axis) is calculated to:

$$N_{fi,Rd,z} = \chi_z \cdot N_{fi,pl,Rd}$$

where:

χ_z	Reduction factor depending on buckling curve c and non-dimensional slenderness
$N_{fi,pl,Rd}$	Design value of the plastic resistance to axial compression in the fire situation

To use the simple calculation model, different constraints have to be fulfilled. Additionally, the column should be part of a braced frame.

Section 4.3.5.2

Table 1. Constraints for using simple calculation model

Constraint	Existing	
$\max l_\theta = 13.5 \cdot b = 13.5 \cdot 0.3 = 4.05$ m	$l_\theta = 0.5 \cdot 4.0 = 2.0$ m	✓
230 mm $\leq h \leq$ 1100 mm	$h = 300$ mm	✓
230 mm $\leq b \leq$ 1100 mm	$b = 300$ mm	✓
$1\% \leq A_s / (A_c + A_s) \leq 6\%$	$19.6/(731.4 + 19.6) = 0.03 = 3\%$	✓
max R 120	R 60	✓
$l_\theta < 10 \cdot b \text{ if } \begin{cases} 230 \leq b < 300 \text{ or} \\ h/b > 3 \end{cases}$	$b = 300$ mm $h/b = 300/300 = 1$	✓

2.2.2 *Calculation of the plastic design resistance and the effective flexural stiffness*

According to Annex G of prEN 1994 Part 1-2, the cross-section of the composite column is reduced. Some parts of the cross-section are reduced by reducing the cross-sectional area and some by reducing the yield stress and modulus of elasticity.

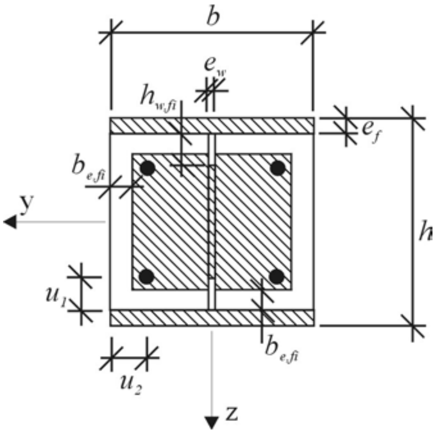


Figure 3. Reduced cross-section for structural fire design

The flanges of the steel profiles are reduced by determining reduction factors for the yield stress and the modulus of elasticity. For this, the average temperature of the flanges has to be calculated.

$$\theta_{f,t} = \theta_{o,t} + k_t \cdot A_m / V$$

Section G.2

The temperature $\theta_{o,t}$ and the reduction factor k_t are given in Table 2. The section factor is calculated as below:

$$\frac{A_m}{V} = \frac{2\cdot(h+b)}{h\cdot b} = \frac{2\cdot(0.3+0.3)}{0.3\cdot 0.3} = 13.3 \text{ m}^{-1}$$

Table 2. Parameters for calculating the average flange temperature (see prEN 1994 Part 1-2, Annex G, Table G.1)

Standard fire resistance	$\theta_{o,t}$ [°C]	k_t [m°C]
R 30	550	9.65
R 60	680	9.55
R 90	805	6.15
R 120	900	4.65

For R 60, the average temperature arises to:

$$\theta_{f,t} = 680 + 9.55\cdot 13.3 = 807 \text{ °C}$$

With this temperature, the reduction factors $k_{y,\theta}$ and $k_{E,\theta}$ are given in Table 3.2 of prEN 1994 Part 1-2, where intermediate values can be interpolated linearly.

$$k_{y,\theta} = 0.06 + \left((900-807)/(900-800)\right)\cdot(0.11-0.06) = 0.107$$

$$k_{E,\theta} = 0.0675 + \left((900-807)/(900-800)\right)\cdot(0.09-0.0675) = 0.088$$

The plastic axial design resistance for the flanges and its flexural stiffness are determined to:

$$N_{fi,pl,Rd,f} = 2\cdot\left(b\cdot e_f\cdot k_{y,\theta}\cdot f_{ay,f}\right)/\gamma_{M,fi,a} = 2\cdot(30\cdot 1.9\cdot 0.107\cdot 23.5)/1.0$$
$$= 286.65 \text{ kN}$$

$$(EI)_{fi,f,z} = k_{E,\theta}\cdot E_{a,f}\cdot\left(e_f\cdot b^3\right)/6 = 0.088\cdot 21{,}000\cdot\left(1.9\cdot 30^3\right)/6$$
$$= 1.58\cdot 10^7 \text{ kNcm}^2$$

The web is reduced by its cross-sectional area and by the yield stress. The reduction of the height is calculated as below, where this height is reduced at both edges of the flange.

$$h_{w,fi} = 0.5\cdot\left(h-2\cdot e_f\right)\cdot\left(1-\sqrt{1-0.16\cdot\left(H_t/h\right)}\right)$$ Section G.3

The parameter H_t is given in Table 3.

Table 3. Parameter for reduction of the web (see prEN 1994 Part 1-2, Annex G, Table G.2)

Standard fire resistance	H_t [mm]
R 30	350
R 60	770
R 90	1100
R 120	1250

Therefore, $h_{w,fi}$ is calculated to:

$$h_{w,fi} = 0.5\cdot(30-2\cdot 1.9)\cdot\left(1-\sqrt{1-0.16\cdot(77/30)}\right) = 3.04 \text{ cm}$$

The yield stress is reduced to:

$$f_{ay,w,t} = f_{ay,w}\cdot\sqrt{1-0.16\cdot\left(H_t/h\right)} = 23.5\cdot\sqrt{1-0.16\cdot(77/30)} = 18.04 \text{ kN/cm}^2$$

The axial design resistance and flexural stiffness for the web during fire exposure are:

$$N_{fi,pl,Rd,w} = \left[e_w \cdot \left(h - 2 \cdot e_f - 2 \cdot h_{w,fi}\right) \cdot f_{ay,w,t}\right] \big/ \gamma_{M,fi,a}$$

$$= \left[1.1 \cdot \left(30 - 2 \cdot 1.9 - 2 \cdot 3.04\right) \cdot 18.04\right] \big/ 1.0$$

$$= 399.26 \text{ kN}$$

$$\left(EI\right)_{fi,w,z} = \left[E_{a,w} \cdot \left(h - 2 \cdot e_f - 2 \cdot h_{w,fi}\right) \cdot e_w^3\right] \big/ 12$$

$$= \left[21{,}000 \cdot \left(30 - 2 \cdot 1.9 - 2 \cdot 3.04\right) \cdot 1.1^3\right] \big/ 12$$

$$= 0.0047 \cdot 10^7 \text{ kNcm}$$

An exterior layer of concrete with a thickness $b_{c,fi}$ is neglected in the calculation. This thickness is given in Table 4.

$\Rightarrow$ $b_{c,fi} = 1.5$ cm Section G.4

Table 4. Thickness reduction of concrete (see prEN 1994 Part 1-2, Annex G, Table G.3)

Standard fire resistance	$b_{c,fi}$ [mm]
R30	4.0
R 60	15.0
R 90	$0.5 \cdot \left(A_m/V\right) + 22.5$
R 120	$2.0 \cdot \left(A_m/V\right) + 24.0$

The rest of the concrete is reduced by the reduction factor $k_{c,\theta}$ which depends on the temperature of the concrete. The average temperature of the concrete is given in Table 5. It depends on the section factor A_m/V.

Table 5. Average temperature of the concrete depending on the section factor (see prEN 1994 Part 1-2, Annex G, Table G.4)

R 30		R 60		R 90		R 120	
A_m/V [m^{-1}]	$\theta_{c,t}$ [°C]	A_m/V [m^{-1}]	$\theta_{c,t}$ [°C]	A_m/V [m^{-1}]	$\theta_{c,t}$ [°C]	A_m/V [m^{-1}]	$\theta_{c,t}$ [°C]
4	136	4	214	4	256	4	265
23	300	9	300	6	300	5	300
46	400	21	400	13	400	9	400
---	---	50	600	33	600	23	600
---	---	---	---	54	800	38	800
---	---	---	---	---	---	41	900
---	---	---	---	---	---	43	1000

$$\Rightarrow \quad \theta_{c,t} = 400 - \left(\left(21 - 13.3\right)/\left(21 - 9\right)\right) \cdot \left(400 - 300\right) = 336 \text{ °C}$$

where:

$$A_m/V = 13.3 \text{ m}^{-1},$$

The reduction factor $k_{c,\theta}$ and the strain $\varepsilon_{cu,\theta}$ corresponding to $f_{c,\theta}$ are given in Table 3.3 of prEN 1994 Part 1-2.

$$k_{c,\theta} = 0.75 + \left(\left(400 - 336\right)/\left(400 - 300\right)\right) \cdot \left(0.85 - 0.75\right) = 0.814$$

$$\varepsilon_{cu,\theta} = \left[10 - \left(\left(400 - 336\right)/\left(400 - 300\right)\right) \cdot \left(10 - 7\right)\right] \cdot 10^{-3} = 8.08 \cdot 10^{-3}$$

The secant modulus of concrete can therefore be calculated to:

$$E_{c,sec,\theta} = k_{c,\theta} \cdot f_c / \varepsilon_{cu,\theta} = 0.814 \cdot 2.5 / \left(8.08 \cdot 10^{-3}\right) = 251.9 \text{ kN/cm}^2$$

The axial design resistance and the flexural stiffness of the concrete can now be determined:

$$\begin{aligned} N_{fi,pl,Rd,c} &= 0{,}86 \cdot \left(\left(\left(h - 2 \cdot e_f - 2 \cdot b_{c,fi}\right) \cdot \left(b - e_w - 2 \cdot b_{c,fi}\right)\right) - A_s\right) \\ &\quad \cdot f_{c,\theta} / \gamma_{M,fi,c} \\ &= 0{,}86 \cdot \left(\left(\left(30 - 2 \cdot 1{,}9 - 2 \cdot 1{,}5\right) \cdot \left(30 - 1{,}1 - 2 \cdot 1{,}5\right)\right) - 19{,}6\right) \\ &\quad \cdot \left(0{,}814 \cdot 2{,}5\right) / 1{,}0 \\ &= 1017{,}3 \text{ kN} \end{aligned}$$

$$\begin{aligned} (EI)_{fi,c,z} &= E_{c,sec,\theta} \cdot \left(\left(\left(h - 2 \cdot e_f - 2 \cdot b_{c,fi}\right) \cdot \left(\left(b - 2 \cdot b_{c,fi}\right)^3 - e_w^3\right) / 12\right) - I_{s,z}\right) \\ &= 251{,}9 \cdot \left(\left(\left(30 - 2 \cdot 1{,}9 - 2 \cdot 1{,}5\right) \cdot \left(\left(30 - 2 \cdot 1{,}5\right)^3 - 1{,}1^3\right) / 12\right) - 1960\right) \\ &= 0{,}909 \cdot 10^7 \text{ kNcm}^2 \end{aligned}$$

The reinforcing bars are only reduced by its yield stress and modulus of elasticity. The reduction factor $k_{y,t}$ for the reduction of the yield stress is given in Table 6 and the reduction factor $k_{E,t}$ for the reduction of the modulus of elasticity is gained from Table 7. Both are depending on the fire resistance class and the geometrical average u of the axis distances of the reinforcing bars to the outer borders of the concrete.

$$u = \sqrt{u_1 \cdot u_2} = \sqrt{50 \cdot 50} = 50 \text{ mm}$$

Section G.5

where:

u_1 the axis distance from the outer reinforcing bar to the inner flange edge

u_2 the axis distance from the outer reinforcing bar to the concrete surface

Table 6. Reduction factor $k_{y,t}$ for the yield stress f_{sy} of the reinforcing bars (see prEN 1994 Part 1-2, Annex G, Table G.5)

Standard fire resistance	u [mm] 40	45	50	55	60
R 30	1	1	1	1	1
R 60	0.789	0.883	0.976	1	1
R 90	0.314	0.434	0.572	0.696	0.822
R 120	0.170	0.223	0.288	0.367	0.436

Table 7. Reduction factor $k_{E,t}$ for the modulus of elasticity E_s of the reinforcing bars (see prEN 1994 Part 1-2, Annex G, Table G.6)

Standard fire resistance	u [mm] 40	45	50	55	60
R 30	0.830	0.865	0.888	0.914	0.935
R 60	0.604	0.647	0.689	0.729	0.763
R 90	0.193	0.283	0.406	0.522	0.619
R 120	0.110	0.128	0.173	0.233	0.285

$\Rightarrow \quad k_{y,t} = 0.976$

$k_{E,t} = 0.689$

The plastic design resistance and the flexural stiffness of the reinforcing bars are calculated to:

$$N_{fi,pl,Rd,s} = A_s \cdot k_{y,t} \cdot f_{sy} / \gamma_{M,fi,s} = 19,6 \cdot 0,976 \cdot 50,0/1,0 = 956,5 \text{ kN}$$

$$(EI)_{fi,s,z} = k_{E,t} \cdot E_s \cdot I_{s,z} = 0,689 \cdot 21\,000 \cdot 1960 = 2,836 \cdot 10^7 \text{ kNcm}^2$$

The design resistance all-over the cross-section is determined to:

$$\begin{aligned} N_{fi,pl,Rd} &= N_{fi,pl,Rd,f} + N_{fi,pl,Rd,w} + N_{fi,pl,Rd,c} + N_{fi,pl,Rd,s} \\ &= 286,7 + 399,3 + 1017,3 + 956,5 \\ &= 2659,8 \text{ kN} \end{aligned}$$

To calculate the effective flexural stiffness of the cross-section, reduction coefficients $\varphi_{i,\theta}$ have to be determined. They are given in Table 8.

Table 8. Reduction coefficients for calculation of effective flexural stiffness (see prEN 1994 Part 1-2, Annex G, Table G.7)

Standard fire resistance	$\varphi_{f,\theta}$	$\varphi_{w,\theta}$	$\varphi_{c,\theta}$	$\varphi_{s,\theta}$
R 30	1.0	1.0	0.8	1.0
R 60	0.9	1.0	0.8	0.9
R 90	0.8	1.0	0.8	0.8
R 120	1.0	1.0	0.8	1.0

$$\begin{aligned} (EI)_{fi,eff,z} &= \varphi_{f,\theta} \cdot (EI)_{fi,f,z} + \varphi_{w,\theta} \cdot (EI)_{fi,w,z} + \varphi_{c,\theta} \cdot (EI)_{fi,c,z} + \varphi_{s,\theta} \cdot (EI)_{fi,s,z} \\ &= 0,9 \cdot 1,58 \cdot 10^7 + 1,0 \cdot 0,0047 \cdot 10^7 + 0,8 \cdot 0,909 \cdot 10^7 + 0,9 \cdot 2,836 \cdot 10^7 \\ &= 4,70 \cdot 10^7 \text{ kNcm}^2 \end{aligned}$$

2.2.3 *Calculation of the axial buckling load at elevated temperature*

Section G.6

The Euler buckling load or elastic critical load follows by:

$$N_{fi,cr,z} = \pi^2 \cdot (EI)_{fi,eff,z} / l_\theta^2 = \pi^2 \cdot 4,70 \cdot 10^7 / (0,5 \cdot 400)^2 = 11610,7 \text{ kN}$$

where:

l_θ	buckling length of the column in the fire situation

The non-dimensional slenderness is obtained from:

$$\bar{\lambda}_\theta = \sqrt{N_{fi,pl,R} / N_{fi,cr,z}} = \sqrt{2659,8/11610} = 0,48$$

where:

$N_{fi,pl,R}$	the value $N_{fi,pl,Rd}$ with partial safety factors $\gamma_{M,fi,I} = 1.0$

The reduction factor χ_z is determined by using buckling curve c of Table 5.5.2 of prEN 1993 Part 1-1 and the non-dimensional slenderness in the fire situation.

prEN 1993-1-1

Section 6.3.1.2

$$\chi_z = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}_\theta^2}} = \frac{1}{0,68 + \sqrt{0,68^2 - 0,48^2}} = 0,86$$

where:

$$\begin{aligned} \Phi &= 0,5 \cdot \left(1 + \alpha \cdot (\bar{\lambda}_\theta - 0,2) + \bar{\lambda}_\theta^2\right) = 0,5 \cdot \left(1 + 0,49 \cdot (0,48 - 0,2) + 0,48^2\right) \\ &= 0,68 \end{aligned}$$

The buckling design resistance is calculated to:

prEN 1994-1-2

$$N_{fi,Rd,z} = \chi_z \cdot N_{fi,pl,Rd} = 0{,}86 \cdot 2659{,}8 = 2287{,}4 \text{ kN}$$

Verification:

$$N_{fi,d} / N_{fi,Rd,z} = 1143{,}8/2287{,}4 = 0{,}50 < 1 \quad \checkmark$$

2.3 *Verification using tabulated data method*

The verification using tabulated data is to be done in the strength domain.

Section 4.2.3.3

When determining the load level $\eta_{fi,t}$ the reinforcement ratio should be between 1% and 6%. Higher or lower ratios should not be taken into account.

$$\frac{A_s}{A_c + A_s} \begin{cases} \geq 1\% \\ \leq 6\% \end{cases}$$

$$\frac{19.6}{731.4 + 19.6} = 0.03 = 3\% \begin{cases} > 1\% \\ < 6\% \end{cases}$$

The load level is calculated to:

$$\eta_{fi,t} = E_{fi,d,t} / R_d = N_{fi,d} / N_{Rd} = 1143.8/4130.4 = 0.28$$

The parameters given in Table 2 of Annex A may be interpolated linearly.

Interpolation between lines 1.1 and 2.1:

$$\min b = \min h = 300 - \left(\frac{0.47 - 0.34}{0.47 - 0.28} \right) \cdot \left(300 - 200 \right) = 231.6 \text{ mm}$$

Table 9. Verification of composite column with partially encased steel sections

Minimum	Existing	
$\min e_w / e_f = 0{,}5$	$e_w / e_f = 1{,}1/1{,}9 = 0{,}58$	✓
$\min b = \min h = 200$ mm	$b = h = 300$ mm	✓
$\min u_s = 50$ mm	$u_s = 50$ mm	✓
$\min A_s / (A_c + A_s) = 4\%$	$A_s / (A_c + A_s) = 3\%$	✗

The reinforcement ratio of the composite column is too low. To increase the ratio, reinforcement bars with bigger diameters or multiple reinforcement bars per corner can be applied.

However, the verification by using the simple calculation model could be accomplished successfully. This shows that the "tabulated data" method leads to conservative results.

REFERENCES

EN 1991, *Eurocode 1:Actions on structures – Part 1-2: General actions – Actions on structures exposed to fire*, Brussels: CEN, November 2002

prEN 1993, *Eurocode 3: Design of steel structures – Part 1-1: General rules*, Brussels: CEN

prEN 1994, *Eurocode 4: Design of composite steel and concrete structures – Part 1-2: General Rules – Structural Fire Design*, Brussels: CEN